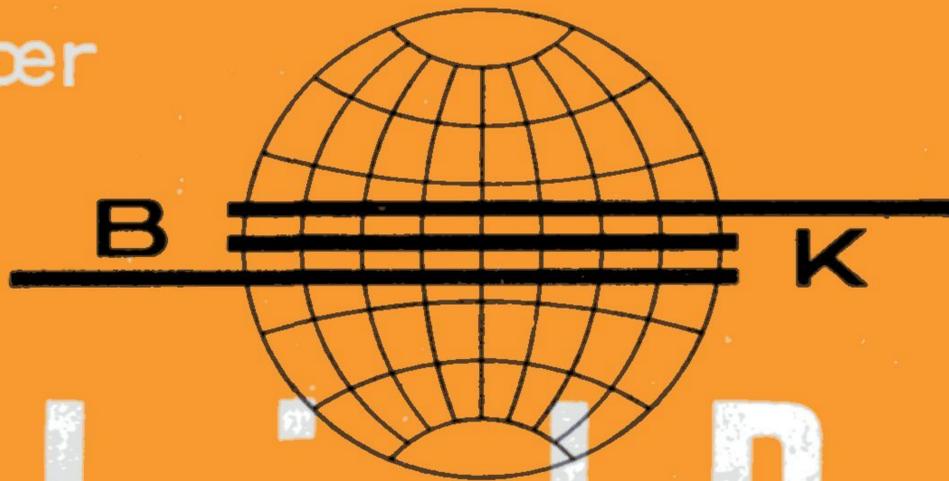
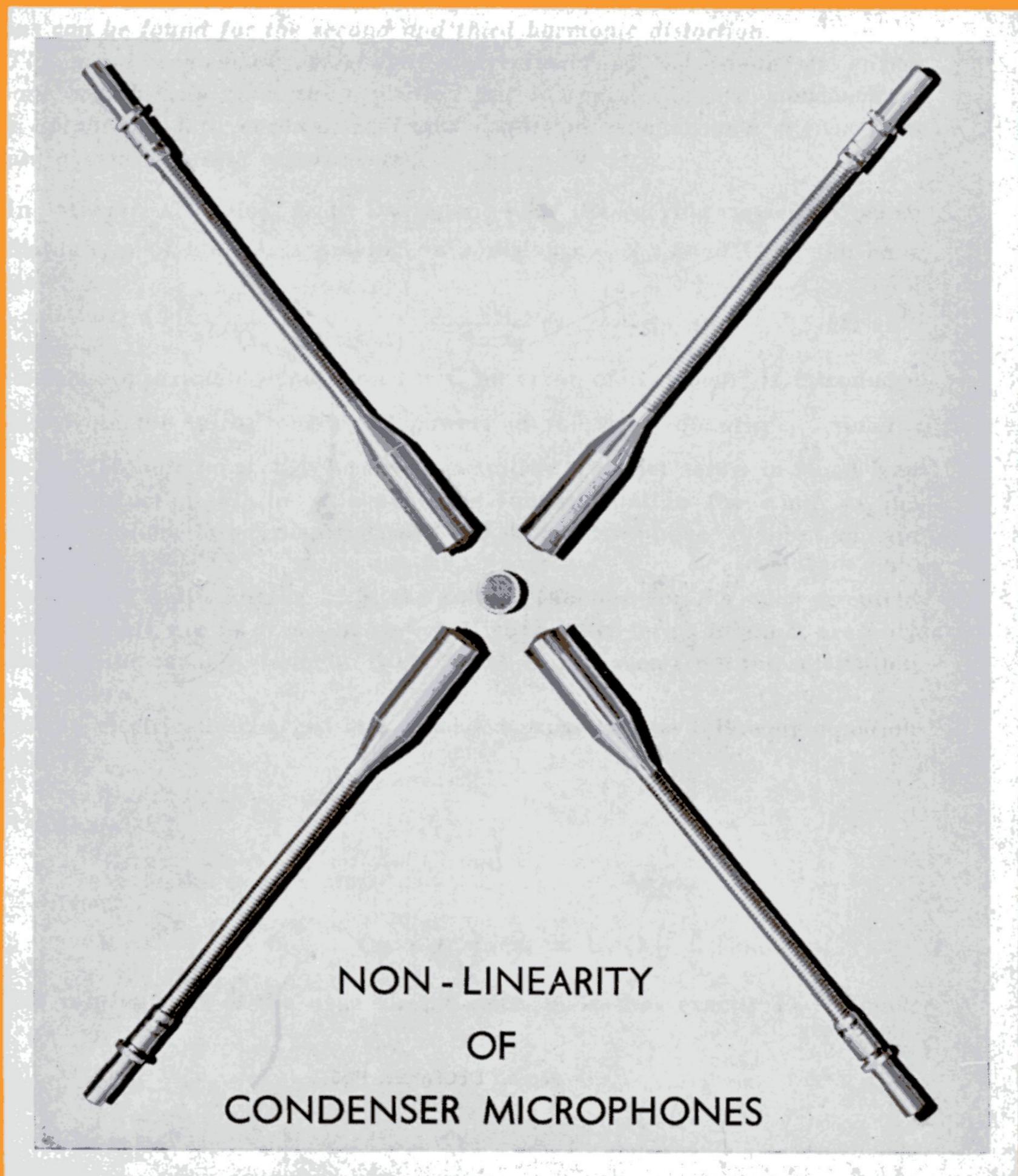


Brüel & Kjøer



# Technical Review

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# NON-LINEARITY OF CONDENSER MICROPHONES

by

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*In the literature concerning this subject there is still a lot of confusion. Even the simplified conception of the condenser microphone, replacing the diaphragm by an equivalent rigid piston, gives rise to some faulty treatments.*

*Experiments, carried out in our laboratory, and described in this article, show that a theory published by W. Ernsthausen in "Archiv für Elektrotechnik" Vol. 31, 1937, p. 487 seems to be most correct. This theory is based on an approximate solution of the differential equation governing the operation of loaded condenser microphones, and relatively simple expressions can be found for the second and third harmonic distortion.*

*The usual, somewhat oversimplified treatment of the condenser microphone, which only takes the distortion due to the mechanical movement of the diaphragm into account, indicates distortion values much higher than those observed during experiments.*

In "Modern Acoustics" A. H. Davis writes for the varying capacity  $C$  when a diaphragm of area  $A$  is moving on a distance  $x_0 + x \sin \omega t$  from the back plate

$$C = \frac{A}{4\pi (x_0 + x \sin \omega t)} = \frac{A}{4\pi x_0} \left(1 - \frac{x}{x_0} \sin \omega t\right) \quad (1)$$

In this approximate expression for  $C$  an error of  $\left(\frac{x}{x_0} \sin \omega t\right)^2$  is introduced (apart from the third, fourth etc. powers of the small quantity  $\left(\frac{x}{x_0} \sin \omega t\right)$ ).

In the footnote on p. 134, however, is stated: "Further terms in  $\sin \omega t$  lead to the production of the octave of the fundamental in the e.m.f. at the terminals when large displacements of the microphone diaphragm are involved".

This is not quite correct, as in the voltage function for the open circuited capacitor there are no terms of  $\sin^2 \omega t$ . Higher order terms in  $\sin \omega t$  are only present in the capacity-function itself, which can be seen from the calculation shown below.

For an electrically charged and unloaded capacitor the following equation holds:

$$E \cdot C = E_0 C_0 = q_0$$

where

$$C = \frac{A}{4\pi (x_0 + x \sin \omega t)} \quad \text{and} \quad C_0 = \frac{A}{4\pi x_0}$$

Then

$$E = \frac{E_0 C_0}{A} 4\pi (x_0 + x \sin \omega t) = E_0 \left(1 + \frac{x}{x_0} \sin \omega t\right) \quad (2)$$

The varying part of the open circuit e.m.f.,  $e$ , is thus exactly  $E_0 \frac{x}{x_0} \sin \omega t$ .

However, taking the second order term into account the capacity function can be written:

$$C = C_0 \left[ 1 - \frac{x}{x_0} \sin \omega t + \left( \frac{x}{x_0} \sin \omega t \right)^2 \right]$$

or

$$C = C_0 \left[ 1 + \frac{1}{2} \left( \frac{x}{x_0} \right)^2 - \frac{x}{x_0} \sin \omega t - \frac{1}{2} \left( \frac{x}{x_0} \right)^2 \cos 2\omega t \right]$$

According to this the distortion due to the second harmonic should be:

$$d_2 = \frac{\frac{1}{2} \left( \frac{x}{x_0} \right)^2}{\frac{x}{x_0}} \times 100 \% = 50 \frac{x}{x_0} \% \quad (3)$$

or with peak open-circuit voltage  $\hat{e} = E_0 \frac{x}{x_0}$ :

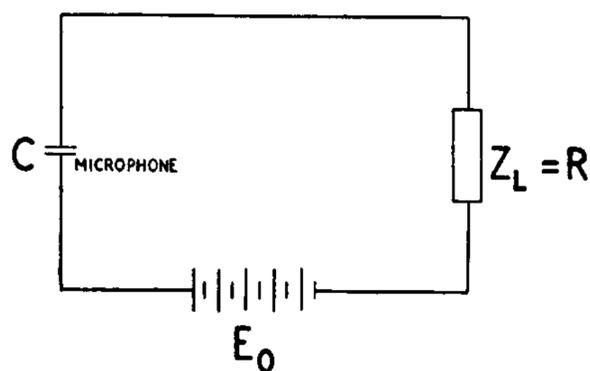
$$\boxed{d_2 = 50 \frac{\hat{e}}{E_0} \%} \quad (4)$$

where  $E_0$  is the polarization voltage. This expression is sometimes found in the literature treating condenser microphones.

The error made in the calculations which lead to (4) is obvious:

The voltage  $E$  is not proportional to the capacity function  $C$ , but is inversely proportional to this function. Formula (4) is consequently not correct.

The 2nd harmonic distortion in the voltage applied to the cathode follower tube under actual working condition of the microphone, is first of all dependent on the load. However, as soon as a load  $R$ , which is not infinite, is introduced in the circuit, the assumption that  $EC = E_0 C_0$  is not valid any more and consequently the concept of a distortionless generator can not be used as a basis for analysis of the problem.



*Fig. 1. Basic circuit diagram for a condenser microphone.*

Starting with the basic circuit diagram for a condenser microphone (fig. 1), and assuming the load to be a pure ohmic resistance the following equation is found:

$$E_0 = R \frac{dq}{dt} + \frac{q}{c} \quad (5)$$

Introducing the capacity function equation (5) becomes:

$$E_0 C_0 = RC_0 \frac{dq}{dt} + (1 + \beta \sin \omega t) q$$

$$\text{where } \beta = \frac{x}{x_0}$$

A solution to this differential equation is given by W. Ernsthausen in "Archiv für Elektrotechnik", and can be written in the form:

$$q = q_c + \beta q_1 \sin(\omega t + \varphi_1) + \beta^2 q_2 \sin(2\omega t + \varphi_2) + \beta^3 q_3 \sin(3\omega t + \varphi_3) \quad (6)$$

where

$$q_c = \text{constant (average charge on the capacitor)}$$

$$q_1 = \frac{E_0 C_0}{\sqrt{1 + (\omega RC_0)^2}}$$

$$q_2 = \frac{E_0 C_0}{2\sqrt{[1 + (\omega RC_0)^2] [1 + (2\omega RC_0)^2]}}$$

$$q_3 = \frac{E_0 C_0}{4\sqrt{[1 + (\omega RC_0)^2] [1 + (2\omega RC_0)^2] [1 + (3\omega RC_0)^2]}}$$

The voltage applied to the grid of the cathode follower is:

$$e = Ri = R \frac{dq}{dt} \quad (7)$$

Differentiation of equation (6) and calculation of the second harmonic distortion gives:

$$d_2 = 100 \times \beta \times \frac{1}{\sqrt{1 + (2\omega RC_0)^2}} \% \quad (8)$$

Similarly for the third harmonic distortion:

$$d_3 = 100 \times \frac{3}{4} \times \beta^2 \frac{1}{\sqrt{[1 + (2\omega RC_0)^2] [1 + (3\omega RC_0)^2]}} \quad (9)$$

An expression for  $\beta$  can be found from the voltage amplitude function of the fundamental frequency:

$$e_1 = R \frac{dq_1}{dt} = \omega R \beta q_1$$

$$\beta = \frac{e_1}{\omega R E_0 C_0} \sqrt{1 + (\omega RC_0)^2} = \frac{e_1}{E_0} \frac{\sqrt{1 + (\omega RC_0)^2}}{\omega RC_0}$$

With increasing frequency  $\beta \rightarrow \frac{e_1}{E_0}$

To check formulae (4) and (8) distortion measurements have been carried out on a Condenser Microphone Type 4111; the sound pressures necessary being estimated in the following way:

According to (4) 1% distortion should be obtained for an open circuit peak voltage:

$$\hat{e} = d_2 \times \frac{E_0}{50} = 1 \times \frac{150}{50} = 3 \text{ volts,}$$

when a polarization voltage of 150 V is employed.

The attenuation of the cathode follower is approx.  $2\frac{1}{2}$  db and the output voltage from the cathode follower should thus be:

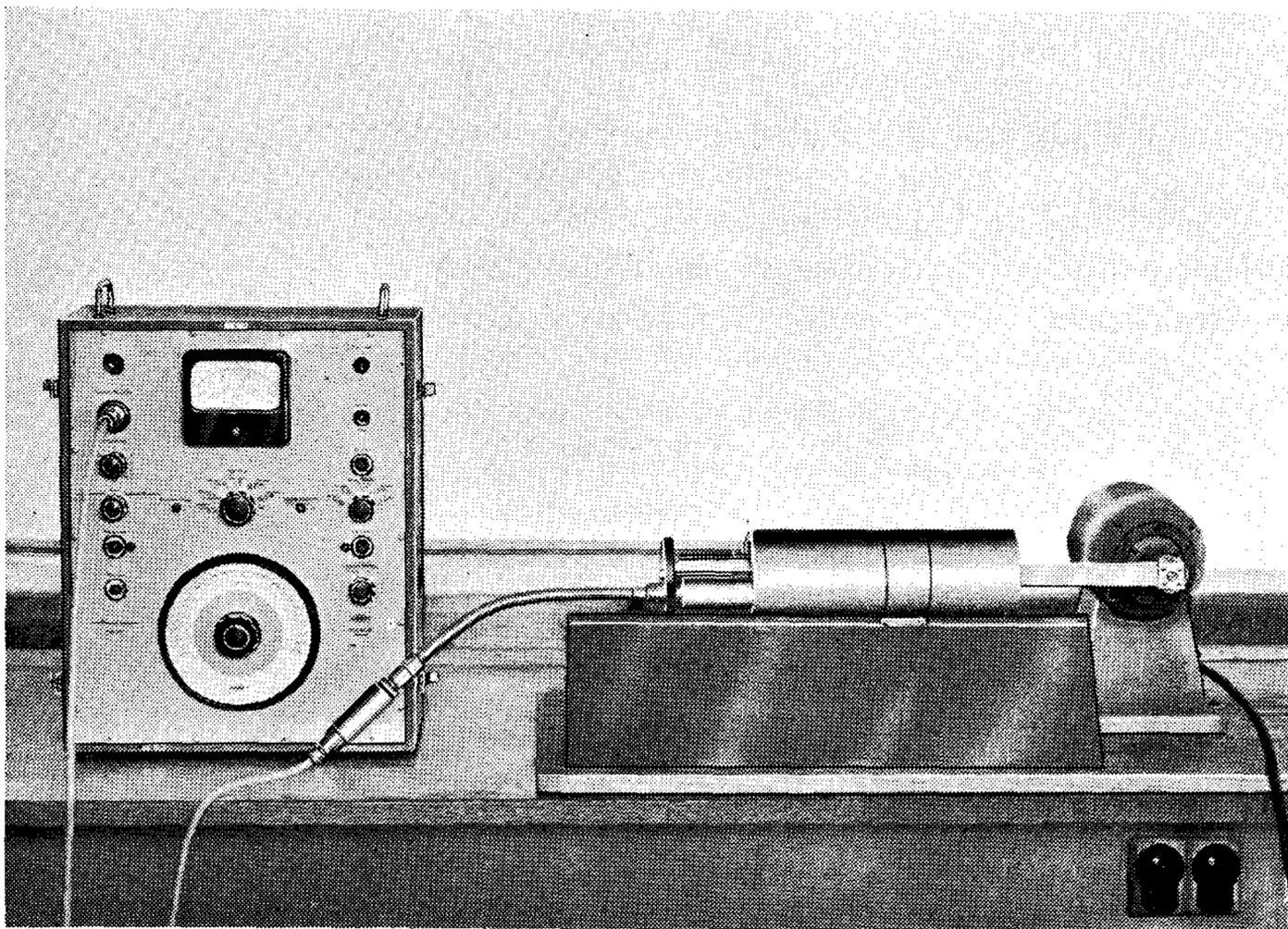
$$e_{\text{out}} = \frac{3}{\sqrt{2} \times 1.33} = 1.6 \text{ volts (r.m.s.)}$$

To obtain a distortion of 4% a voltage of 6.4 volts r.m.s. should be measured across the output terminals of the cathode follower.

The sound pressures necessary to obtain these output voltages can then be calculated from the sensitivity of the microphone, which is approx. 2 mV/ $\mu$ bar.

1.6 volts r.m.s. then corresponds to a sound pressure level of 0.8 millibar (or 132 db re  $2 \times 10^{-4}$   $\mu$ bar r.m.s.), 6.4 volts to a sound pressure level of 3.2 millibar (or 144 db re  $2 \times 10^{-4}$   $\mu$ bar).

The distortion of the Condenser Microphone Type 4111 was measured in two different set-ups.



*Fig. 2. Measuring arrangement for distortion measurements. Fundamental frequency 50 c/s (Big pistonphone).*

First the distortion at a fundamental frequency of 50 c/s was measured in an arrangement as shown in fig. 2.

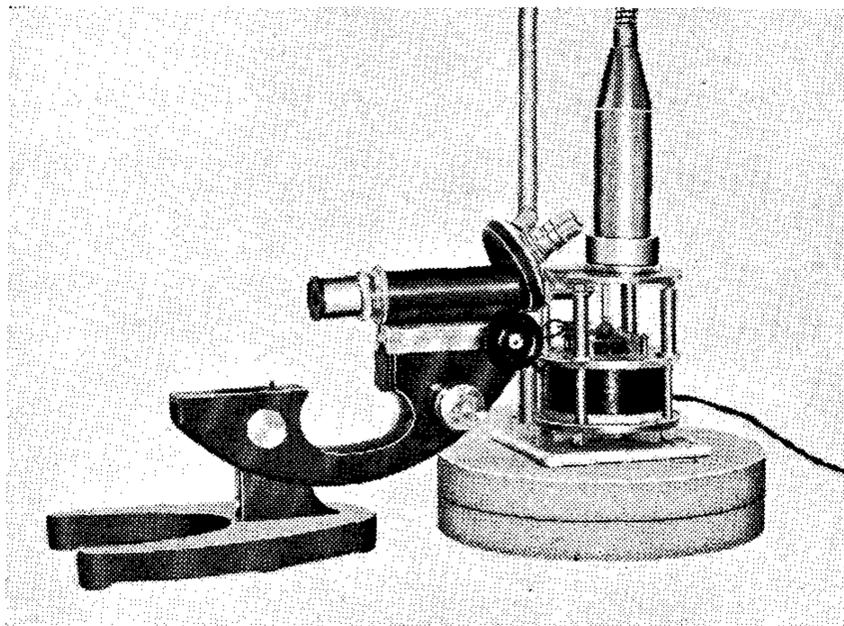
An a.c. motor is driving a piston of 42 mm diameter in a cylinder block placed in a tube of 102 mm inner diameter. Both cylinder block and tube can be moved in order to obtain different air volumes between the piston and the terminal plate at the left, where the microphone is mounted.

With two different strokes  $s$  of 1.5 and 5.0 mm, and the length  $l$  of the cavity adjustable between 2 and 20 cm, sound pressures can be obtained between 1 and 40 millibar r.m.s., i.e. between 134 and 166 db re  $2 \times 10^{-4} \mu\text{bar}$ . These values are found by employing the normal pistonphone formula

$$p = \eta \times \gamma \times \frac{P_0}{V} \times A_p \times \frac{s}{1.4}$$

where  $\gamma = C_p/C_v$  is the ratio of specific heats for air,  $p_0$  is the atmospheric pressure,  $A_p$  the area and  $s/1.4$  the r.m.s. value of the piston stroke. The factor  $\eta$  is dependent upon whether, at the frequency under consideration, the compression and rarefaction occur adiabatically ( $\eta = 1$ ) or isothermally ( $\eta = 1/\gamma = 0.71$ ). With the volume big in comparison with the wall area,  $\eta$  is assumed to be 1, as the constant wall temperature forces the process to occur adiabatically.

The distortion of the piston movement itself, caused by the finite length (32 cm) of the piston arm, was graphically determined. With  $s = 1.5$  mm the total distortion was less than 0.5 %, with  $s = 5.0$  mm less than 1.7 %. The larger of the two strokes was therefore only applied when the distortion was exceeding 3 %; the length  $l$  was first small to attain high pressure levels inside the tube.



*Fig. 3. Measuring arrangement for distortion measurements. Fundamental frequency higher than 50 c/s. (Small pistonphones).*

Another arrangement, used at frequencies above 50 c/s is shown in fig. 3. A piston of 15 mm in diameter is moving in a very smooth grinded, exactly fitting cylinder, which, in its turn fits the coupler of the Microphone Calibration Apparatus, Type 4119.

With this set-up sound pressures between 0.01—10 millibar r.m.s. were obtained for frequencies below 200 c/s. For higher frequencies (up to 800 c/s) the highest obtainable sound pressure was only about 1 mbar, due to the smaller amplitude of the piston movement.

At sound pressures lower than 0.6 millibar the accuracy was reasonable. The cavity was then 26 mm in diameter and 5.2 mm high, and the length of the double stroke  $20 \mu$  ( $f < 200$  c/s).

Higher sound pressures were obtained by decreasing the cavity volume (diameter 15 mm, height 1 mm for 10 millibar).

Due to leakage, and to the inaccuracy of the factor  $\eta$  in the applied pistonphone formula at small volumes, the accuracy decreased with increasing sound pressure.

The distortion was measured by means of the Audio Frequency Spectrometer Type 2109.

With the bigger of the two pistonphones, excited at the frequency 50 c/s, (which is the mid-frequency of the 2nd filter of 2109) the harmonics up to the 6th harmonic inclusive, can be determined. The small pistonphone can be driven from the B. F. O. Type 1012 (or 1014) at any of the mid frequencies 50, 64, 80, 100 and 125 etc. of the Spectrometer filters thus allowing measurement of harmonics up to the 6th inclusive to be taken. For distortions smaller than 1 % at 50 c/s and smaller than 0.5 % at 100 c/s, the Frequency and Distortion Measuring Bridge Type 1602 was used as rejection filter for the fundamental frequency.

By accurate tuning of the Bridge an attenuation of 120 db can be obtained.

The input of the Distortion Bridge was connected to the jack marked "Filter Input" on 2109, thus employing only the first, low distortion amplifier ( $d_{tot} < 0.1 \%$ ), of this instrument. The different components of the harmonic distortion were measured with the Frequency Analyzer, Type 2105, at the output of 1602.

With a cathode follower of conventional design, see fig. 4, the dependency of the distortion upon the parameters, R, C and  $\omega$  according to formula (8) was verified.

Due to the practical construction of condenser microphone the value of

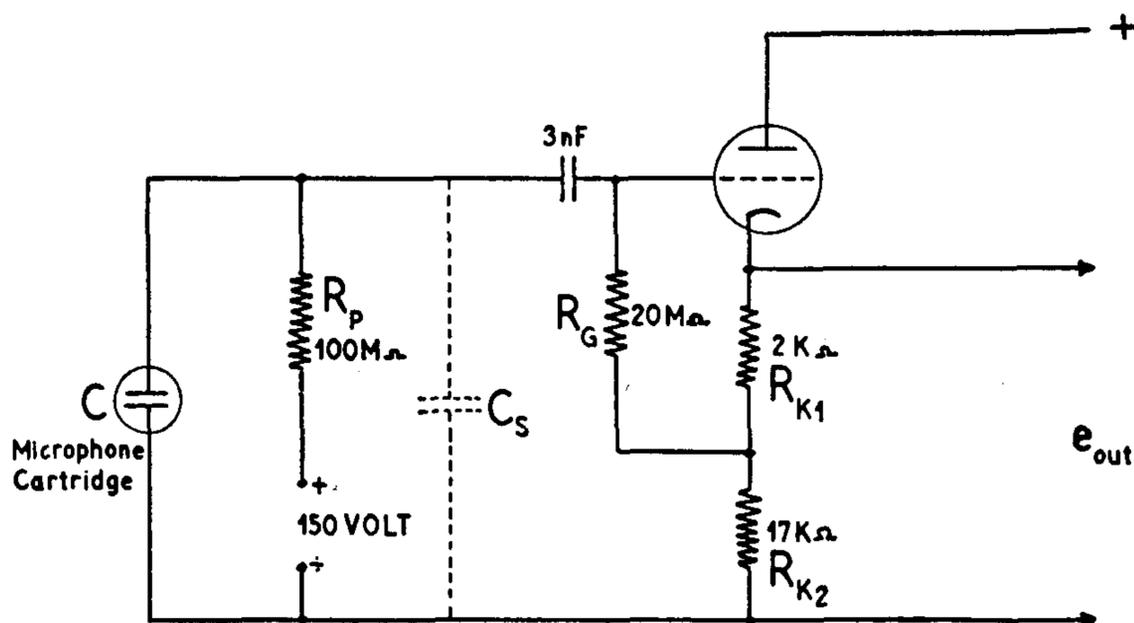


Fig. 4. Circuit diagram for cathode follower of conventional design.

$C_0$  is determined not only by the active part of the cartridge, (i.e. the capacitance between the diaphragm and the "back plate"), but depends also upon the stray capacitance of the cartridge, and on the circuit wiring.

The value of  $R$  is given by the parallel combination of  $R_p$  and the effective input impedance of the tube.

According to fig. 4 the latter is:

$$R_{in} \approx R_g \left( 1 + \frac{R_{k_2} S_g}{1 + R_{k_1} S_g} \right)$$

With a transconductance of 1.85 mA/V the input impedance should then amount to about 150 M $\Omega$ .

However, for the actual verification, only the product  $RC$  has to be known. This quantity was determined from an accurate measurement of the lower limiting frequency  $f_0$  (the frequency at which the response is 3 db down).

Expressed in the quantity  $f_0$  the formulae (8) and (9) become:

$$d_2 = 100 \times \beta \frac{1}{\sqrt{1 + (2f/f_0)^2}} \% \quad (10)$$

$$d_3 = d_2 \times \beta \times \frac{3}{4} \times \frac{1}{\sqrt{1 + (3f/f_0)^2}} \% \quad (11)$$

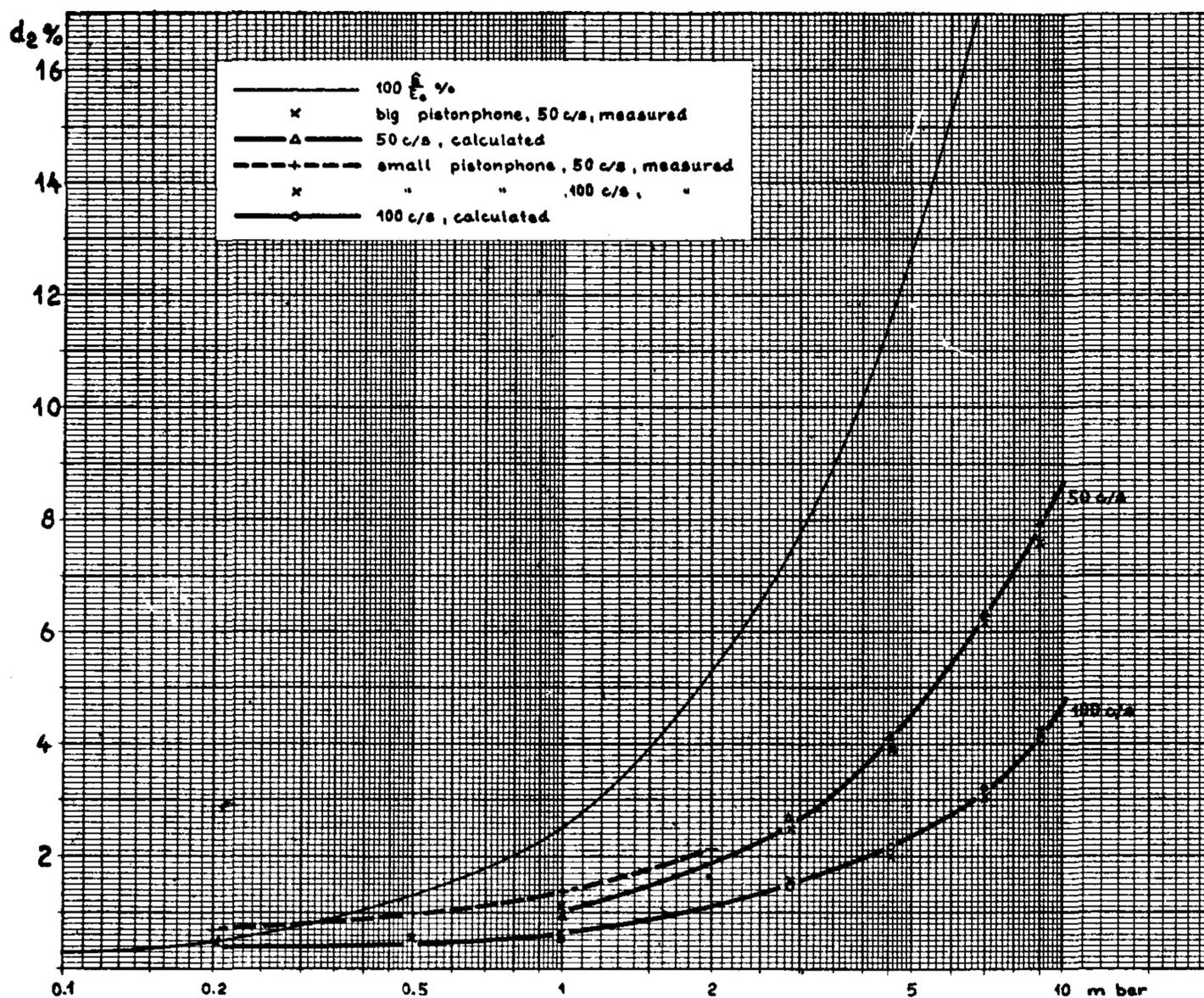


Fig. 5. Distortion curves obtained from measurements at sound pressures up to 10 m/bar.

Fig. 5 indicates the distortion for sound pressure levels up to about 10 mbar r.m.s., or 154 db, with an arbitrary cartridge mounted on the cathode follower of fig. 4.

At 50 c/s the distortion was measured in both the big pistonphone, and the small pistonphone. In the latter set-up the distortion was also measured a 100 c/s. The values according to (4) are indicated with the thinner line.

The value  $\hat{e}$  of formula (4) is found as  $1.4 \times$  the B.F.O. voltage which, supplied as generator between ground and the cartridge, gives the same output as was obtained with the sound pressure applied during the experiment. For this purpose the cartridge housing is insulated from ground by means of a bakelite ring.

For instance, with a sound pressure of 6.9 mbar the microphone output voltage was 12 V, the second harmonic 0.74 V and the third harmonic 0.20 V. The equivalent BFO voltage to yield 12 V output via the shielded cartridge was 19 V, and the expression  $100 \hat{e}/E_0$  consequently 17.7 %. The measured lower limiting frequency was 38 c/s and with the aid of formulae (10) and (11) the 2nd harmonic distortion is found to be 6.3 % and the third harmonic distortion 2 %. The measured distortions were 6.2 and 1.7 % respectively. The calculated values are also shown in fig. 5.

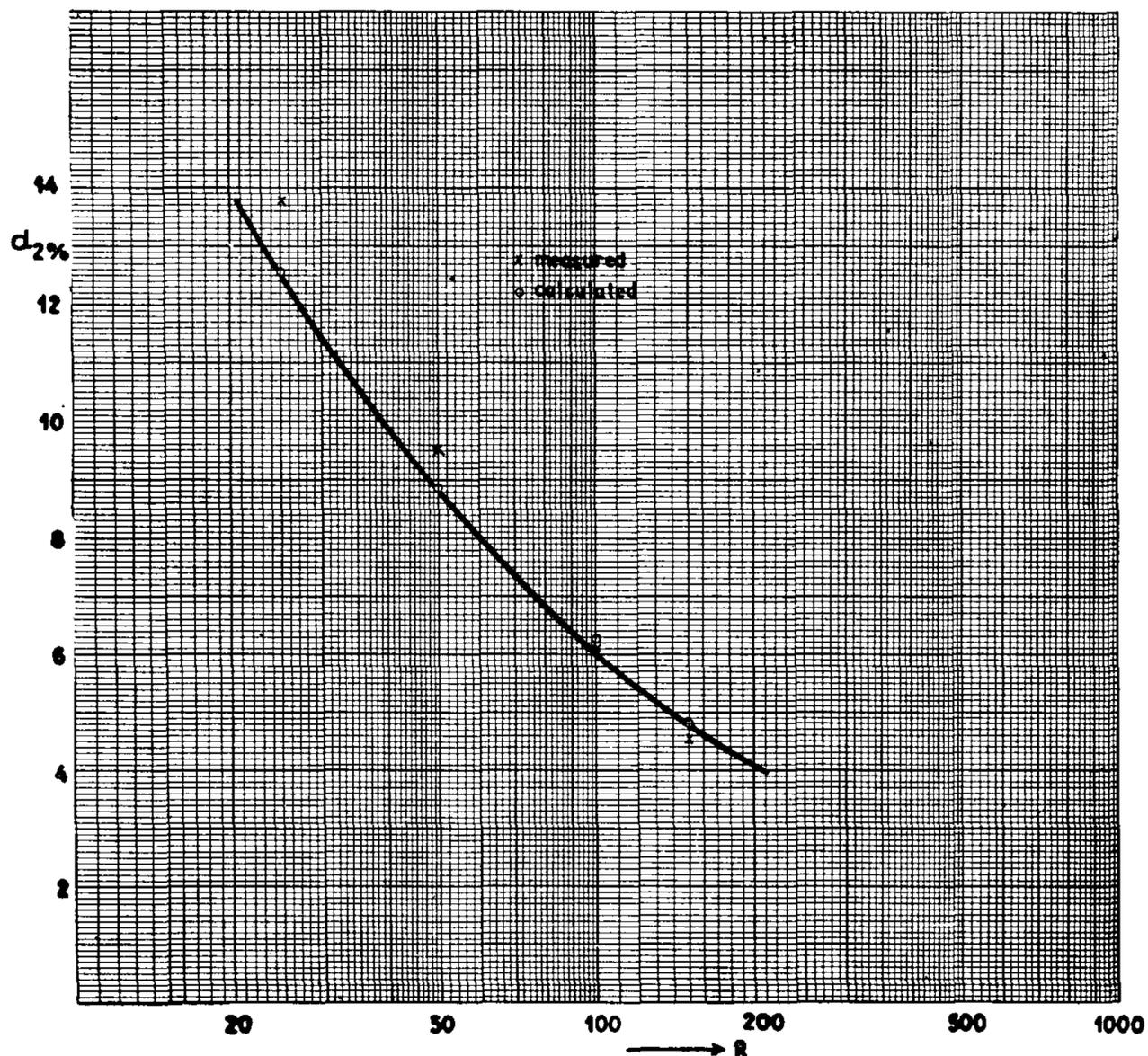


Fig. 6. Dependency of the distortion upon R. Sound pressure 6,9 m/bar.

It can be seen from the curves that to obtain a total distortion of 4% at frequencies higher than 100 c/s the sound pressure level must exceed 154 db re  $2 \times 10^{-4}$   $\mu$ bar. According to the formula (4) a distortion of 4% should already be present with a sound pressure level of 144 db.

With a constant sound pressure of 6.9 mbar the dependency of the distortion upon the parameter R was verified by changing the resistance  $R_p$ . The normal value of  $R_p$  is 100 M $\Omega$ . This was changed to 25, 50 and 150 M $\Omega$  respectively. For each value the corresponding lower limiting frequency was determined, and with the aid hereof the distortion calculated. The result is shown in fig. 6, where the line through the circles depicts the calculated distortion.

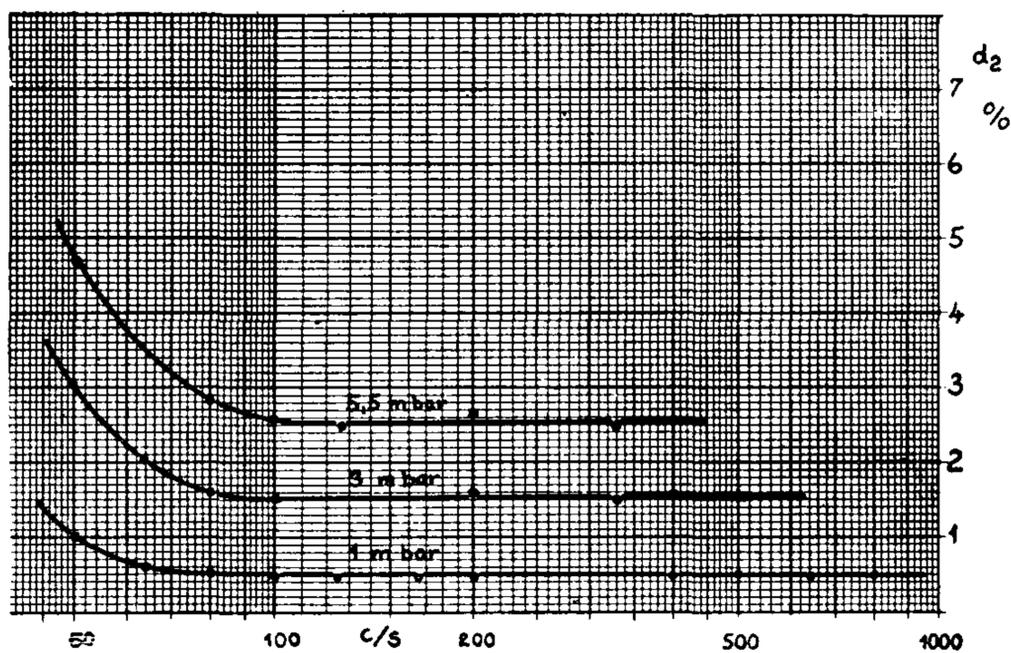


Fig. 7. Distortion curves as measured on the small pistonphone.

Fig. 7 indicates the distortion as measured on the small pistonphone. At frequencies up to 100 c/s the results are in good agreement with those given in fig. 5. However, for frequencies higher than 100 c/s the measurements show that other sources of distortion than the circuit distortion as treated at p. 5 start to play a role. The distortion measurements carried out at frequencies up to 800 c/s for a sound pressure of 1 mbar, and up to 400 c/s for a sound pressure of 5.5 mbar, show a tendency to yield a constant distortion for frequencies higher than 100 c/s. These results indicate that the second distortion source is not frequency- but only pressure dependent. However, the total distortion due to both influences, still is much less than can be calculated from formula (4).

The Condenser Microphone Type 4111 is equipped with a cathode follower of a slightly different design than the one shown in fig. 4.

To extend the linear response of the microphone, an a.c. connection is made between the resistor  $R_p$  and the cathode. The impedance, as seen from the microphone cartridge, is thereby increased, and a lower limiting frequency of 15 c/s is obtained instead of 38 c/s.

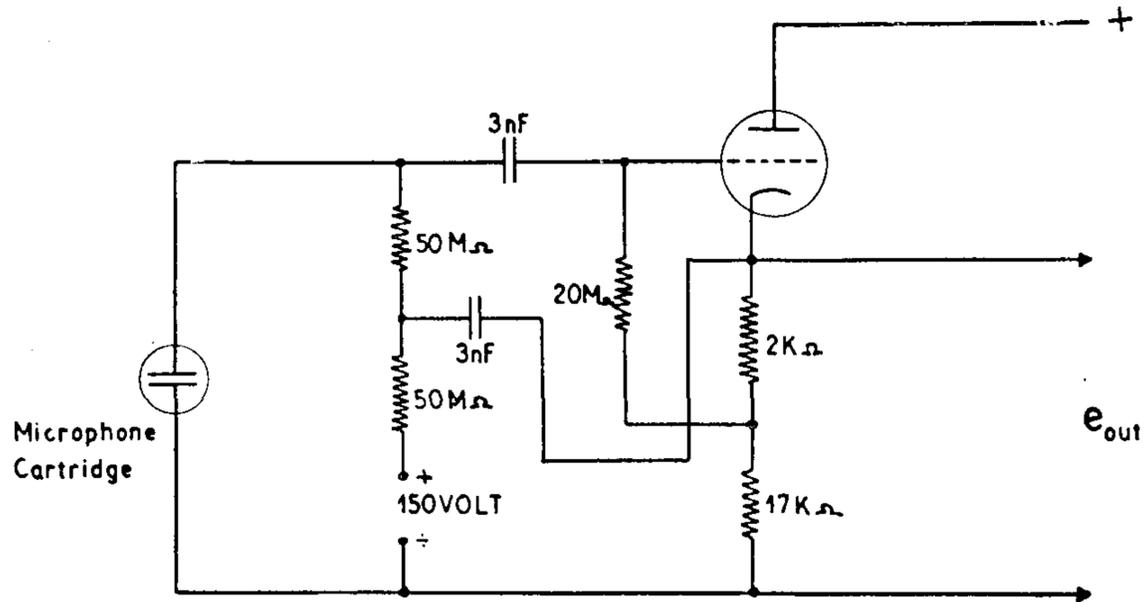


Fig. 8. Circuit diagram of the cathode follower employed in Condenser Microphone Type 4111.

Furthermore the second harmonic distortion is reduced, as can be seen from the curves shown in fig. 9. The measurements were carried out with the big pistonphone measuring arrangement.

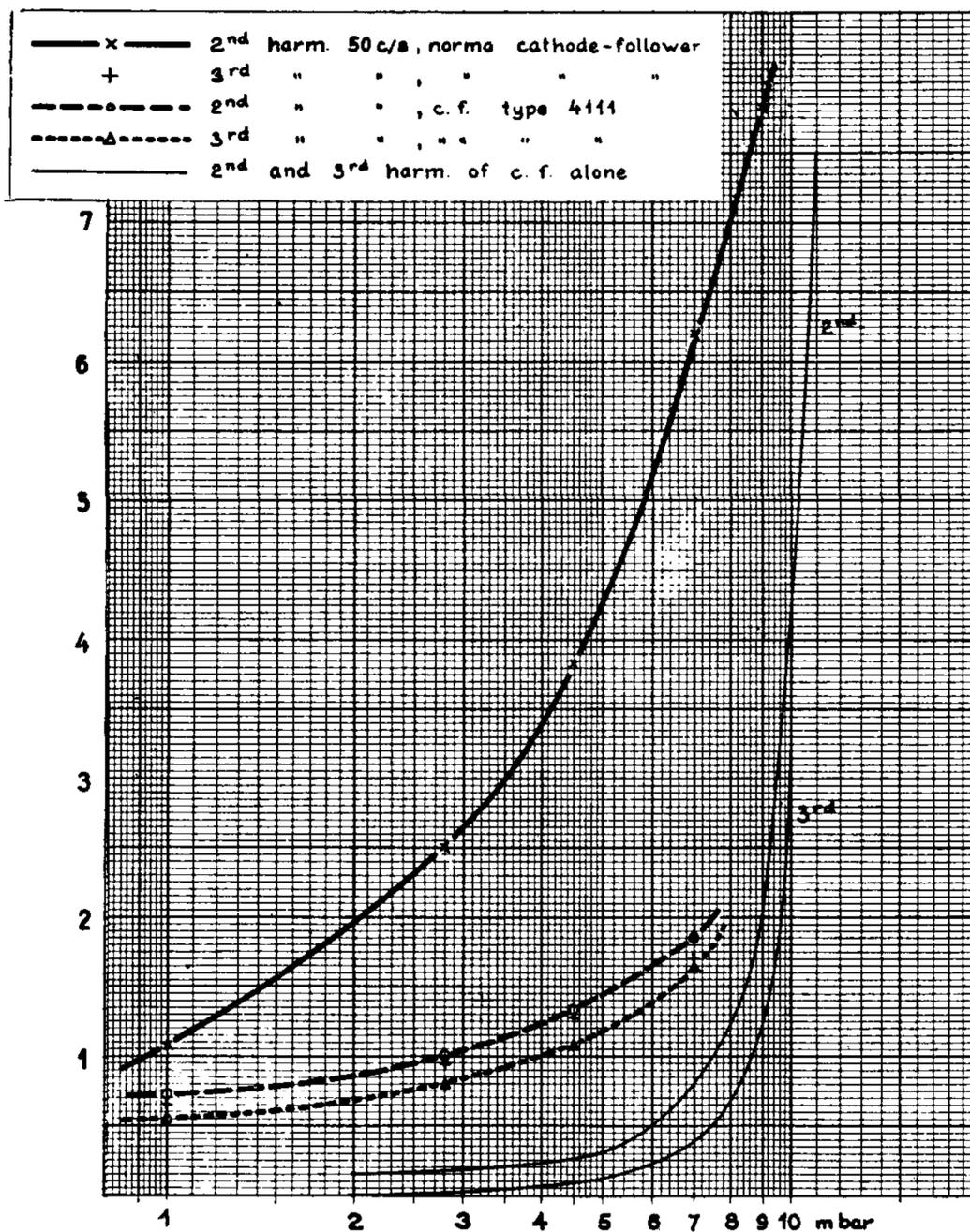


Fig. 9. Distortion curves for a Condenser Microphone Type 4111.

The thin lines represent the cathode follower's own 2nd and 3rd harmonic distortion. The distortion of the cathode follower is here given as a function of the sound pressure, valid for the particular cartridge under consideration, but is of course only dependent upon the input voltage applied to the cathode follower. At about 20 volts the circuit is overloaded and the distortion rises quickly.

The conversion of input voltage to sound pressure was made by taking the sensitivity of the cartridge at 50 c/s into account. This was, for the cartridge employed, rather high: 2.1 mV/ $\mu$ bar.

The distortion due to the cathode follower itself will therefore be large for sound pressures higher than approx. 10 mbar, which consequently sets a limit to sound pressures at which the microphone should be used.

As a last experiment the second harmonic distortion was measured with different cartridges on the microphone. The result is shown in fig. 10.

It can be seen from the described theory and experiments that the

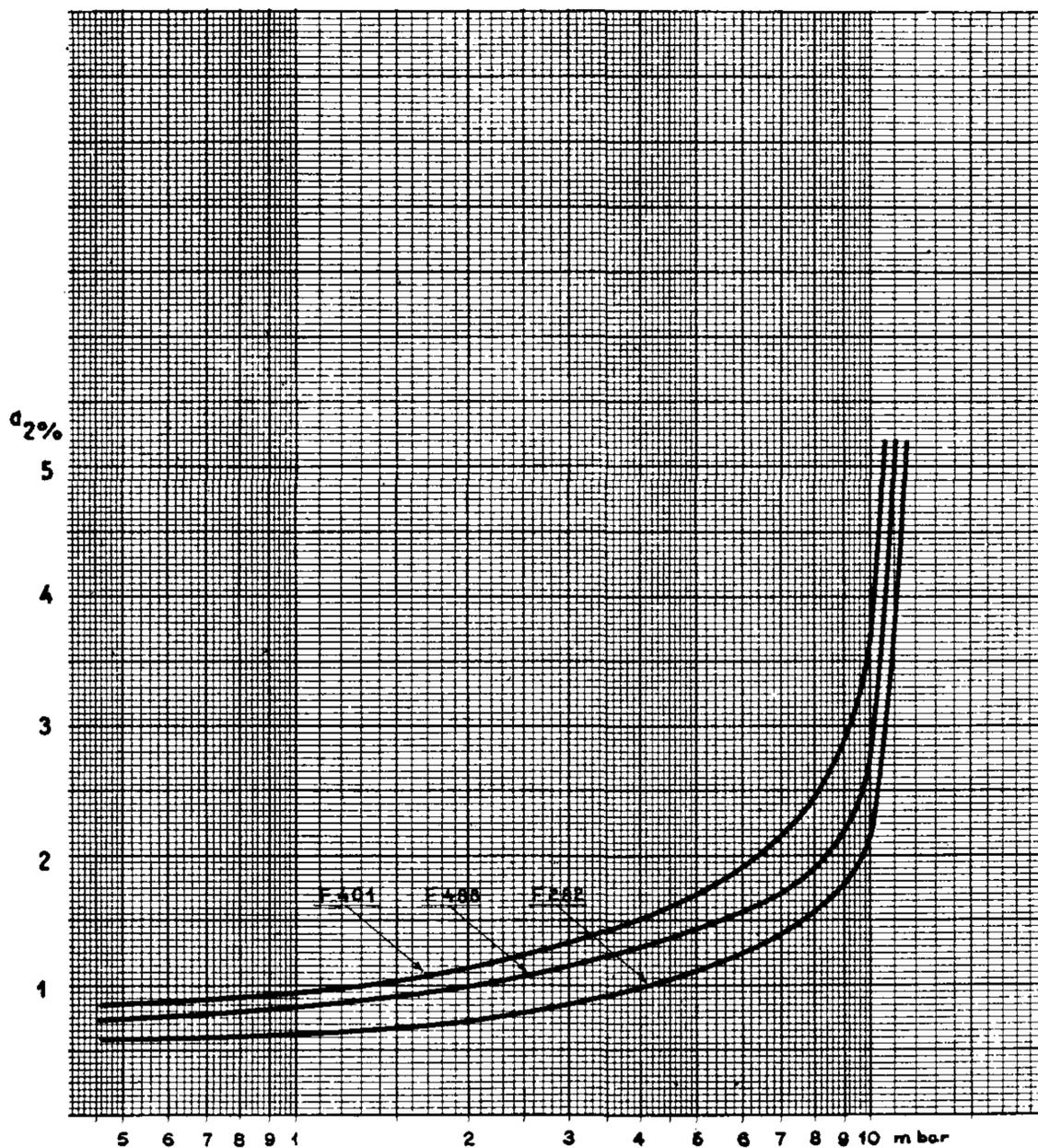


Fig. 10. Measurement of second harmonic distortion employing different cartridges on the microphone.

harmonic distortion in condenser microphones actually is smaller than found from the formula

$$d_2 = 50 \times \frac{\hat{e}}{E_0}$$

The dependency upon frequency and sound pressure shows that a large amount of harmonic distortion will only be present at very low frequencies and at high sound pressures, so that, at normal operating conditions, this type of distortion will be almost negligible.



Aktieselskabet

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