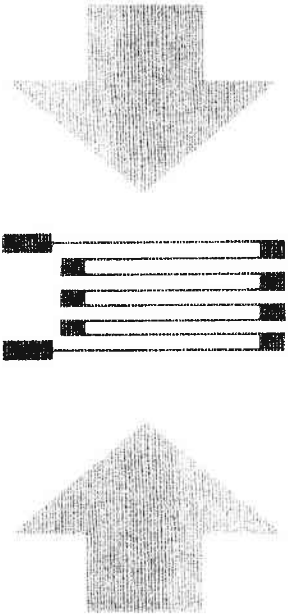

Application of B&K Equipment to

Strain Measurements

Brüel & Kjær

Application of B & K Equipment

to

STRAIN MEASUREMENTS

by

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October 1975

ISBN 87 87355 08 6

CONTENTS

FOREWORD

The object of this publication is to give the prospective strain gauge user a simple explanation of the basic theory and practice of measurement using strain gauges. It will show how various factors, such as type of gauge, gauge material, measuring conditions and type of instrumentation can influence the results obtained.

Elementary stress and strain theory will be outlined and the working principle of the strain gauge described. Various types of gauge and measuring arrangement will be discussed, and some practical tips given for gauge handling so that the user can get the best from a measuring arrangement. Adhesives for strain gauge mounting will be discussed. Alternative applications for strain gauges, as stress, force, pressure, or torque transducers will also be mentioned.

The Brüel and Kjær program of strain measurement instrumentation will be introduced, and guidance given to help in the choice of gauge and system for particular applications. As an additional aid to the prospective user, information is given on various sources of gauges and adhesives.

The following letters and symbols have been used throughout this book:

a	resistance ratio
A	Amperes
C	degrees Centigrade
d	differential sign
E	excitation voltage and Youngs Modulus
F	degrees Fahrenheit
g	acceleration due to gravity
G	Modulus of Rigidity
i	current
k	gauge factor
l	length
N	error factor due to input resistance of measuring instrument
P	error factor due to lead resistance
r	relative change in resistance
R	resistance
t	temperature
T	error factor due to bridge non-linearity
V	voltage
x, y, z	perpendicular axes
α	angle
β	angle
δ	small increment
δR	change in resistance
δl	change in length
δk	change in gauge factor
ϵ	strain
θ_t	temperature expansion coefficient
μ	Poissons Ratio
$\mu\epsilon$	microstrain
π	circle constant
σ	stress
τ	shear stress
Ω	ohms

CHAPTER 1
INTRODUCTION

In the early days, machinery and structures were developed and built on a trial and error basis, making extensive use of the "rule of thumb" approach to the problems of design. This "rule" worked very successfully in areas of slow technological development, where principles and methods were unsophisticated. A new part would be designed using very conservative safety factors, constructed, possibly tested before being put into service, and then modified by "beefing-up" or lightening as experience indicated. With the advent of mass production and the demand for high speed machinery and really efficient construction, these older design methods were no longer technically or economically viable. This was a particular problem in the newly developing aircraft industry, to take just one example, where massive safety factors could not be used, as every unit of weight had to be deployed to give the maximum advantage. Further, innovation in the new industry came so rapidly that technical developments soon outpaced the ability of the older empirical methods to resolve the problems that arose. It soon became impossible to perform all the calculations necessary to design aircraft parts having maximum strength and minimum weight without employing teams of mathematicians, incurring costly time penalties, or risking technical obsolescence before the development was completed. What was needed was a thorough knowledge of the actual load and stress distribution, which in turn called for accurate determination of the strains ($\propto$ stresses) present, so that the calculation and design process could be accelerated without becoming unreliable.

This climate led to the evolution of new methods of calculation to be used in stress analysis, and to the development of several new instruments for determining strain experimentally, under actual service conditions. Earlier methods of strain measurement employed direct mechanical measurement on the part, often supplemented by optical amplification to overcome the difficulty of detecting minute changes in length with the unaided eye. Later, electrical instrumentation was developed that used transducers employing capacitance, inductance or piezoelec-

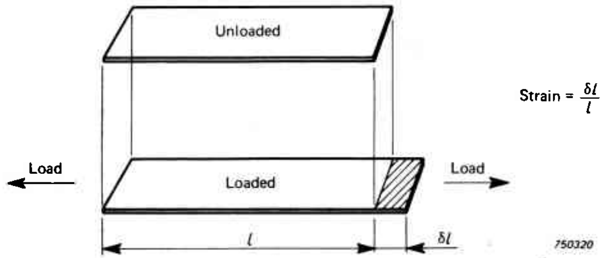

Fig. 1.1. The definition of strain

tric effects. These electrical methods suffered from many of the disadvantages associated with the mechanical extensometers, such as comparatively large size and mass, which could influence the behaviour of the item under test and create clamping difficulties. They also suffered from their own problems, an over-sensitivity to vibration or temperature, complex detection circuitry, a lack of transducer robustness, or a price sufficient to preclude their use in the large numbers necessary for really effective stress determination on complex constructions. Nevertheless, all these measuring methods are still in use, and where conditions are suitable, they yield satisfactory results. More recently, the electrical resistance "strain gauge" has come into widespread use replacing the earlier methods in most applications.

The strain gauge was developed in the late thirties by two researchers in the USA. Working independently of each other, Simmons at CALTEC and Ruge at MIT developed a strain gauge consisting of a length of wire glued to the test object so that changes in length (strains) on the surface are transferred to the wire. These length changes cause altera-

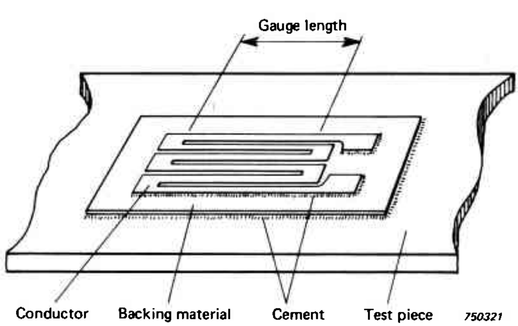

Fig. 1.2. The electrical resistance strain gauge

tions in the resistance of the wire which can be measured by comparatively simple electrical circuitry. The modern strain gauge, examples of which are shown in Fig.1.2 and Fig.1.3, works in exactly the same way, with strain being detected by measuring the resistance variations caused by changes in the gauge length of the wire. The strain gauge can be very small and compact having negligible mass to exert a minimum of influence on the measuring object, and be easily mounted on the test specimen, usually by cementing.

The electrical detection circuits required to measure the very small changes in the gauge resistance are comparatively uncomplicated, being variations of the familiar Wheatstone bridge. When suitable compensation circuits are employed or self-compensating gauges used, temperature sensitivity is virtually eliminated. The resistance strain gauge allows a very economically priced measuring system to be made, where the actual cost per gauge is often so low as to be of virtually no consequence. Gauge costs are no longer a hinderance to the use of strain gauges, cemented in their hundreds on a structure, to solve any particular stress analysis task by actual multiple measurements, instead of the laborious calculation procedures, based on extrapolation from a few measurements, that had been used previously.

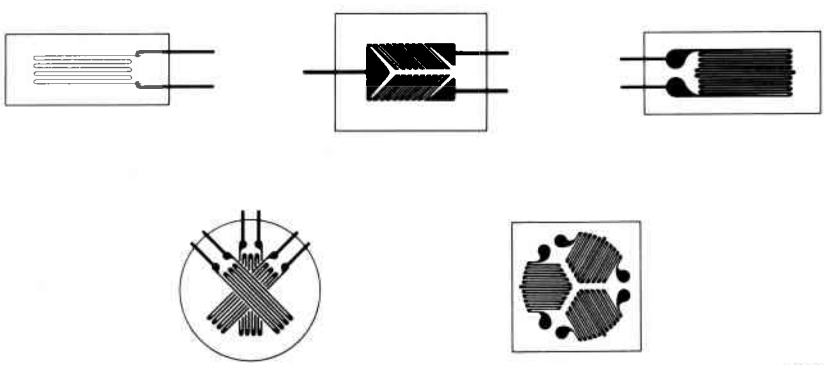

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Fig.1.3. Examples of typical strain gauges

Typical well known applications for strain gauges include experimental strain and stress measurement on aircraft, boats, cars and other forms of transportation. Strain gauges are also used for the measurement of stress in larger structures, for example apartment buildings and office blocks, pressurized containers, bridges, dams, etc. The strain gauge is an important laboratory implement used for pure research, as a design tool in the development stages of many machines and structures, and as a teaching aid to demonstrate basic engineering concepts in educational establishments.

Less well known applications of strain gauges include their use in transducers, where some physical property, about which information is required, will be arranged to deflect a strain gauged member, the amount of deflection being related to the property to be measured. Typical examples of transducer applications for strain gauges are dynamometer rings for force or load measurement, pressure transducers that use a strain gauged diaphragm, and displacement measurement with a thin strain-gauged "feeler".

CHAPTER 2
STRESS AND STRAIN

The maximum benefit from strain gauge measurements can only be obtained when a correctly assembled measuring system is allied with a thorough knowledge of the factors governing the strength and elasticity of materials. This knowledge allows the strain gauges to be deployed in the most effective manner, so that reliable measurements can be obtained that lend themselves to clear unambiguous interpretation.

During the design and construction of machines and structures, the strength of the material to be used plays a very important part in the calculations. The strength of the material is used to find whether the parts can carry the loads demanded of them without excessive deformation or failure. These load carrying abilities are normally characterised in terms of STRESS, that is the amount of load carried by a given area. Therefore stress is quoted in pressure units, force per area (for example Pa = N/m^2, kp/mm^2, lbf/in^2). Similarly, other important criteria for a material, such as the limit of proportionality, the ultimate strength, and the breaking strength, are also usually given in terms of stress. However stress itself cannot be measured, it must be deduced from the mechanical dimensions and the applied load. For example, a known tension load can be applied to a test specimen, and the stress calculated from the applied load and the cross section. The specimen will not change colour as with elevated temperatures, or affect a magnetic needle as with the passage of an electric current. The only physical sign of loading is the mechanical deformation due to the load, and it is this deformation that can be measured by the strain gauge. Hooke showed the relationship between load and extension to be a linear function, therefore if the deflection is measured and this relationship is known, the applied load can be calculated.

In the same way that loads are characterised in terms of stress, extension is characterised in terms of STRAIN, where strain is usually defined as the change in length per unit length (mm/mm, in/in), a dimensionless ratio. A direct relationship known as Young's Modulus (E), similar to Hooke's Law, exists between stress and strain and is shown

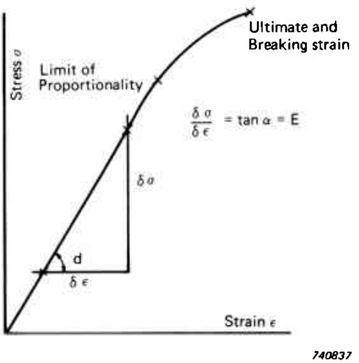


Fig.2.1. Stress/Strain curve for a typical Metal

in Fig.2.1. As strain is dimensionless, E has stress dimensions, force per area.

Taking a general purpose construction steel as an example, the yield point and limit of proportionality lie at a stress level of approximately 26 kp/mm^2 (37000 lbf/in^2), and the ultimate stress is 45 to 50 kp/mm^2 (65000 to 72000 lbf/in^2). Using a value for Young's Modulus of 21 × 10^3 kp/mm^2 (30 × 10^6 lbf/in^2), the yield point strain can be found from,

$$\text{strain} = \frac{26}{21 \times 10^3} = 1240\,\mu\varepsilon$$

It is worth remembering this value as it is typical for a construction type steel.

From the foregoing, it can be seen that if the Young's Modulus of the material is known, and the strain is measured, it should be a simple matter to calculate the stress. This is true when the direction of the stress is known, which is the case when a simple strut is exposed to tension. However, in all other cases there is a further complication, which is examined below.

Fig.2.2 shows the exaggerated deformation of a strut under simple tension loading which causes an increase in length and a corresponding decrease in cross section. This is known as the Poisson Effect, and it means that if strain is measured in either of the planes perpendicular

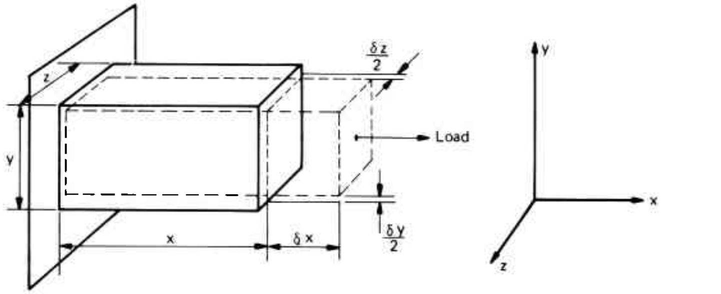


Fig.2.2. Deformation of a strut

to the applied load, a negative strain with lesser magnitude will be detected. The magnitude of this lesser strain depends upon Poisson's Ratio (μ), a material's constant that varies from material to material. It is usually about 0,3, so that the strain measured perpendicular to loading will be approximately —0,3 times the strain measured parallel to the loading. These strains have no associated stress as there is no load applied in their direction.

Simply stated, the relationship

$$E = \frac{Stress}{Strain}$$

is only valid in the direction of the applied load, and not in any other direction.

Fig.2.3 further illustrates the reason by plotting the magnitudes of stress and strain, together with their directions for a single point in the simply loaded strain field.

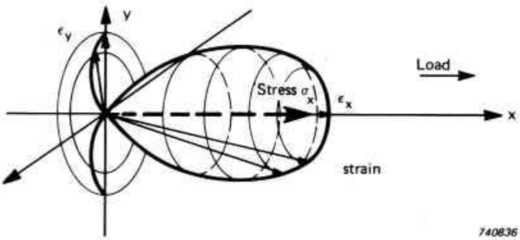


Fig.2.3. Stress and strain about a point in a simply loaded system

Stress lies in the same plane as the loading, and so does the maximum strain, but the minimum strains (ϵ_y, ϵ_z) lie in planes perpendicular to the loading plane. These planes parallel and perpendicular to the direction of loading are the PRINCIPAL PLANES, and the stresses that act upon them are known as the PRINCIPAL STRESSES. It should also be noted that there are no shearing stresses on principal planes.

It will now be appreciated that it is not possible just to multiply the measured strain by the Modulus E to obtain the maximum stress. Additional data on the direction of loading (location of the principal planes) will be necessary, especially where loads are applied in more than one plane.

The problem can be simplified by dividing the information into various components that can be resolved into the three perpendicular axes. The strain in each axis can be considered to be composed of the primary strain due to the loading, and the strain produced by the Poisson Effect. For the example shown in Fig.2.2, the components can be written as follows:

$$\frac{\delta x}{x} = \varepsilon_x = \frac{\sigma_x}{E} \quad : \quad -\frac{\delta y}{y} = \varepsilon_y = -\frac{\mu\sigma_x}{E} \quad : \quad -\frac{\delta z}{z} = \varepsilon_z = -\frac{\mu\sigma_x}{E} \qquad (2{:}1)$$

Similar equations can be written for loads applied in the y and z axes. If now the loads are applied simultaneously in the three axes, the strain in any direction can be found by adding the components algebraically:

$$\begin{aligned} \varepsilon_x &= \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E} \\ \varepsilon_y &= \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_z}{E} \\ \varepsilon_z &= \frac{\sigma_z}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_x}{E} \end{aligned} \qquad (2{:}2)$$

Converting these equations into a form that yields stress, illustrates the point that the stress in any plane is a function of the stress in the other planes:

$$\begin{aligned} \sigma_x &= E\varepsilon_x + \mu\sigma_y + \mu\sigma_z \\ \sigma_y &= E\varepsilon_y + \mu\sigma_x + \mu\sigma_z \\ \sigma_z &= E\varepsilon_z + \mu\sigma_y + \mu\sigma_x \end{aligned} \qquad (2{:}3)$$

so that even when the location of the principal planes is known, measurements must be made in each of the axes.

The practical limitations of strain gauge measurement provide the means of simplifying the expressions in equations 2:3, as normally strain gauges are cemented to the surface of the part being tested (one exception is when gauges are used for measurements on concrete). Stress at a surface cannot act perpendicular to the surface plane so effectively the gauge is measuring a two dimensional strain system. All the terms for one of the axes (for example z) can be eliminated so that the equations for principal stress can be rewritten as follows:

$$\sigma_x = \frac{E}{1-\mu^2}(\varepsilon_x + \mu\varepsilon_y)$$
$$\sigma_y = \frac{E}{1-\mu^2}(\varepsilon_y + \mu\varepsilon_x) \qquad (2{:}4)$$

Often the directions of the principal stresses are not known in the practical measurement situation, so it is advantageous to have expressions for principal strains that refer to arbitrarily positioned axes as shown in equation 2:5.

$$\varepsilon_\alpha = \varepsilon_x \cos^2\alpha + \varepsilon_y \sin^2\alpha + \beta_{xy}\sin\alpha\cos\alpha \qquad (2{:}5)$$

The equation defines the strain ϵ_α at a point, where α is the angle that the strain makes with the X axis in arbitrary X — Y axes. The strain ϵ_α can be measured by a strain gauge, and when measurements are made at three different angles to obtain three values for ϵ_α and α, three equations can be solved simultaneously to give ϵ_x (the strain in direction X), ϵ_y (the strain in direction Y), and β_{xy} (the shearing strain present at the point). Further, when the values obtained are substituted into equation 2:6,

$$\tan 2\alpha_p = \frac{\beta_{xy}}{\varepsilon_x - \varepsilon_y} \qquad (2{:}6)$$

the angle α_p that the principal planes make with the arbitrary axes can be determined. Hence, when α_p is substituted back into equation 2:5 the principal strains and shearing strain at the measuring point can be found and substituted into equation 2:4 to give the stresses acting at the point.

The derivation of equations 2:5 and 2:6 is shown in Appendix 1. Appendix 2 contains worked examples to illustrate how the principal stresses and strains can be calculated from actual gauge measurements, and also demonstrates how Mohr's Strain Circle can also be used to find these values.

The calculation processes detailed in Appendix 2 are sufficiently tedious, especially where many measuring points have to be evaluated, for various researchers to have devised ways to try and simplify them. One common method uses computers for high speed data reduction (sometimes in real-time), this is particularly effective on multi-channel measuring systems. Where fewer measurements have to be evaluated, or when time is of lesser importance, special nomographs have been employed with considerable success.

Nomographs have been devised for several groupings of strain gauges, and for various test sample materials. Common types of nomograph are available for use with special groups of strain gauges that have been arranged at predetermined angles to each other, all on the same backing. The strain gauge groups are known as "Rosette Gauges", and they will be more fully described in Chapter 3. The nomographs can yield the position of the principal axes, magnitudes of the maximum and minimum normal strains, magnitude of the shear strain, and with some types, the stresses can also be determined from the nomograph. Many types of nomograph are based upon the geometry of the Mohr Strain Circle, but as the steps towards a solution differ from nomograph to nomograph, they will not be described further in this book.

However, with the elementary theory of stress and strain described in this chapter, and with the use of Mohr's Strain Circle which is detailed in Appendix 2, the engineer is able to get all the necessary stress information from the measurements made by the most commonly used strain gauge arrangements. The calculations or constructions are straight forward, and the only additional information required is Youngs modulus for the test material, and its Poissons ratio.

CHAPTER 3
THE STRAIN GAUGE

As mentioned in the introduction, the electrical resistance strain gauge consists of a conductor cemented to the test object. The conductor has a very small cross section so that the adhesive cementing it to the specimen is strong enough to hold it securely. This allows the strains to be transmitted from the test object directly to the conductor without relative slip between test object and conductor, or buckling of the conductor under compression. The small changes in the gauge length of the conductor that are caused by a load applied to the test object induce small changes in the resistance of the conductor (an effect first described by Lord Kelvin), and these changes in gauge resistance are detected by the measuring instrumentation. The change in gauge resistance is related to the change in gauge length (strain) by the Gauge Factor k:

$$k = \frac{\delta R}{R} / \frac{\delta l}{l} = \frac{\delta R}{\varepsilon R} \qquad (3:1)$$

where

R = gauge resistance
δR = change in gauge resistance
l = gauge length
δl = change in gauge length
ϵ = strain

One of the major factors that affect the performance and usefulness of any strain gauge is the material from which the conductor is made. Ideally the conductor should have a high Gauge Factor, so that small strains give as large changes as possible to the resistance. The specific resistance of the material should also be high, to give the biggest possible changes in resistance when strained i. e. better resolution. These qualities make the strain gauge sensitive to small strains.

Further, multiples of a given load (strain) must give the same multiple of the resistance change, i. e. the Gauge Factor must be linear, as implied by equation 3:1, and not vary with the degree of loading. Similarly, the Gauge Factor should not vary with time, then repeated applications of a given load will always give the same resistance change.

The conductor material must also be as insensitive to temperature changes as possible to prevent variations in temperature from causing apparent strains that could be as large as the actual strain due to mechanical loading. It should be noted that with modern gauge materials, it is often possible to use the temperature sensitivity of the conductor material to compensate for the temperature sensitivity of the material in the test object. Temperature compensation methods will be described later.

Normally, the choice of a strain gauge conductor material is taken out of the hands of the engineer who is interested in making strain measurements. Gauges are items to be purchased ready-made, and it is unusual (but not unknown) for them to be fabricated on the job. The gauge manufacturer has done the work of materials selection and treatment, and supplies a range of strain gauges with guaranteed characteristics to suit most common measuring situations. The test engineer merely has to select the gauge characteristics that most accurately match the particular measurement requirement.

Wire and Foil Strain Gauges

Earlier types of strain gauge were made of thin copper-nickel or chrome-nickel alloy wire approximately 0,025 mm (0,001 in) in diameter. To achieve the longest practical gauge length, which gives bigger resistance changes, and at the same time the minimum occupied area so that the measurements can approximate point strain determination, the conductor is normally folded into a grid pattern similar to that shown in Fig.3.1. It can be seen that the grid layout still has a long gauge length, but the transverse sensitivity has not been increased by any great amount, however, it still exists and should be taken into account.

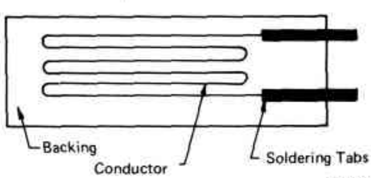


Fig.3.1. Typical wire strain gauge

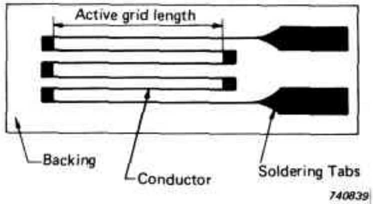


Fig.3.2. Typical foil strain gauge

Considerable savings in the weight of backing material and adhesive are also made with this arrangement.

More recently, the many advantages of the foil gauge have led to its wide-spread use, but it has not completely replaced the wire gauge. In the foil gauge the conductor is made by etching a grid pattern in a thin metal foil only a few micrometres in thickness, made of a similar alloy to the wire, or by cutting the grid from foil using accurate dies. These processes allow accurate and cheap production of almost any conceivable grid pattern, including complex shapes, or very small gauges. A typical foil gauge grid is illustrated in Fig.3.2. The large tabs at each turn of the conductor path make the foil gauge very insensitive to strains across the grid due to the comparatively low resistance in the tabs. Another important feature of the tabs, is that their large surface ensures that linear conditions exist over the complete active length of the grid. End effects are minimised, and the creep problem is also greatly reduced in this way too. End effects are illustrated in the greatly exaggerated drawing shown in Fig.3.3, note that the actual gauge length of the grid is unaffected.

Foil gauges also have a greater ratio of surface area to cross sectional area than wire gauges, which gives them greatly enhanced heat

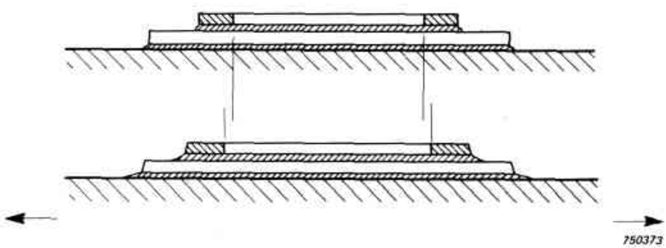


Fig.3.3. Diagram illustrating the end effects

dissipation qualities that permit higher voltage levels to be used for gauge excitation. These have a greater effect on the measuring circuit, and hence much improved resolving power and accuracy. The increased surface area also gives a larger contact area for cementing onto the test object, thus minimising the problem of creep.

Typical values for strain gauges in series production that are available "off the shelf" from suppliers are similar for wire and foil gauges:

Gauge Factor: approximately 2. This is usually quoted individually to two decimal places with the gauge or pack of gauges and often a tolerance, for example ± 1%, will also be given.
Resistance: standardized values, 120 Ω, 350 Ω, 600 Ω, and 1000 Ω. A resistance tolerance is often quoted, for example ± 0,25%.
Linearity: measurements are accurate within 0,1% up to 4000 $\mu\epsilon$, and within 1% up to 10000 $\mu\epsilon$.
Breaking strain: 20000 to 25000 $\mu\epsilon$.
Fatigue life: up to 10^7 strain reversals.
Temperature compensation: normally gauges are available with automatic compensation that matches the temperature expansion coefficient θ_t of one of the three most commonly used construction metals:

General purpose steels with	$\theta_t = 11 \times 10^{-6}$ *per °C (6,1 × 10^{-6} per °F)*
Stainless steels with	$\theta_t = 17 \times 10^{-6}$ *per °C (9,5 × 10^{-6} per °F)*
Aluminium with	$\theta_t = 23 \times 10^{-6}$ *per °C (12,8 × 10^{-6} per °F)*

Some manufacturers also supply gauges compensated for use on titanium, magnesium alloys, or plastic materials. When these are cemented to the material for which the resistance/temperature characteristic has been matched with the expansion coefficient, the apparent strain due to temperature variations on normal gauges can be held down to less than ± 1,5 $\mu\epsilon$/°C (± 0,8 $\mu\epsilon$/°F) over a temperature range from —20° to + 150°C (—5° to + 300°F).

Semiconductor Strain Gauges

In the last decade, semiconductor strain gauges have come into use with Gauge Factors 50 or 60 times greater than for wire or foil gauges. Semiconductor gauges consist of a strip conductor made from a single crystal of silicon or germanium that contains an accurately adjusted amount of impurity to give the characteristic desired.

Semiconductor gauges are more sensitive to temperature variations, and generally not so rugged as wire or foil gauges. They are more suitable for dynamic measurements but can be used for short term measurements of static strain levels. They require special handling to get the best out of them.

Typical values for production semiconductor strain gauges are:

Gauge Factor: 100 (individually calibrated).
Resistance: 120 Ω.
Linearity: measurements are accurate within 1% up to 1000 $\mu\epsilon$.
Breaking strain: approximately 5000 $\mu\epsilon$.
Fatigue life: 10^6 strain reversals.

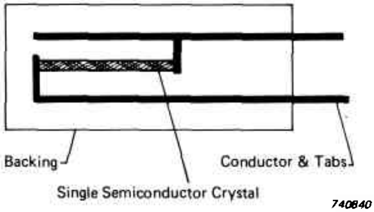


Fig.3.4. Typical semiconductor gauge

At the present time, there is only a very limited selection of semiconductor strain gauges available. This together with their higher price serve to restrict them to more specialized applications where the high Gauge Factor is an advantage, for example in the measurement of very low strain levels. The least strain that can be measured with semiconductor gauges is of the order of 0,001 $\mu\epsilon$, while metal gauges can measure only 0,1 $\mu\epsilon$. Some of the special techniques required for use with semiconductor gauges will be discussed in Chapter 10.

However, it is not really possible to think in terms of one type of general purpose gauge, because many different factors influence the type selected for any particular measurement application. Outside the above mentioned "normal" ranges, other factors like backing materials, or the adhesives employed limit the use of the gauge, particularly with extended temperature ranges. Therefore special gauge types have been developed with conductor characteristics, backing materials, and adhesives tailor-made for more specialized applications.

Special Gauge Types

So far in this chapter, the only strain gauges that have been considered have been single element gauges for measuring linear strain. However, when different grid configurations or multi-grid arrangements are used, improved measurements or additional information can be obtained with a minimum of extra effort.

In Chapter 2 it was shown that when the directions of the principal axes are not known, three gauges can be used to give the necessary information. However, three gauges give almost three times as much labour in preparation and mounting, assuming that there is enough room for them, and a deal of extra calculation, especially when the gauges have not been mounted at an "easy" angle from each other.

To get around these complications as mentioned earlier, strain "rosettes" are often used where two, three, or four measuring grids are mounted on the same backing so that they can all be cemented at the same time on to the test specimen in one easy operation. Strain rosettes have standard, accurately determined 45°, 60°, or 90° angles between the different grids to help simplify the calculation. Charts or calculators are often available for use with standard rosette types, so that strain and the direction of the principal planes can either be read

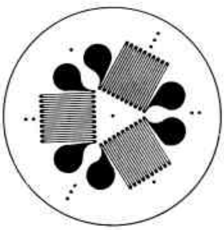


Fig. 3.5a. Delta rosette

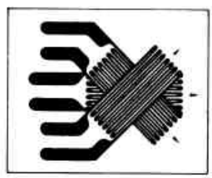


Fig. 3.5b. Stacked grid 90° rosette

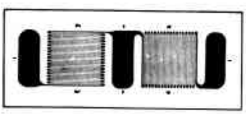


Fig. 3.5c. Two element 90° grid

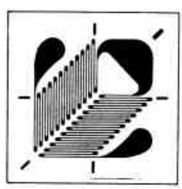


Fig. 3.5d. Herringbone grid

Fig. 3.5. Some typical strain rosettes

directly, or found with a very minimum of simple arithmetic. Fig.3 shows some of the more common strain rosette arrangements.

The Delta rosette in Fig.3.5a is a very common type used to determine the directions of the principal axes, and the principal strains acting on them. A rosette for similar applications is shown in Fig.3.5b, but in this case the gauge grids have been stacked in a sandwich arrangement to save space, and to give a closer approximation to a point strain measurement. When this type of rosette is used with higher excitation voltages, heat dissipation can be a problem because of the close proximity of the grids to each other.

The two element gauge shown in Fig.3.5c has its grids arranged at 90° to each other. This arrangement can be used to augment the measured change in resistance, i. e. give a larger Gauge Factor, when the two grids are connected into adjacent arms of a measuring bridge in the way shown in Fig.6.4. Gauge Factors approximately 1,3 times the normal value can be obtained with this arrangement.

Fig.3.5d shows a two gauge rosette with the grids arranged in a herringbone pattern. This type of gauge, and similar arrangements incorporating four grids at 90° to each other are frequently employed for measuring torsional strains on axles and shafts.

Another type of sandwich construction is that used in the "Flexagage" manufactured by the Budd Company. This gauge is employed to measure bending strains on plates and panels where one surface is inaccessible. The two measuring grids are mounted, one on each surface of a thick backing piece with an accurately known thickness of material between them. When the assembly is cemented to the test panel one gauge is in contact with the panel surface and measures the surface strain. From the measurement obtained by the other grid, it is possible

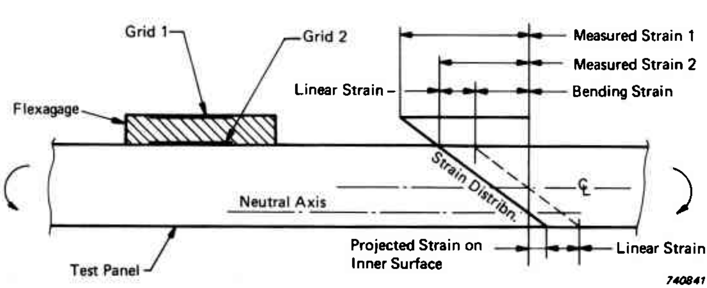


Fig.3.6. Principle of the Flexagage

to deduce the position of the neutral axis, the amount of bending, and the linear strain present. Fig.3.6 illustrates the principle.

A simple geometry gives the position of the neutral axis, while the difference between the strains projected onto the two surfaces of the panel is a measure of the linear strain present. The algebraic average of the projected surface strains gives the bending strain at the panel surface.

The gauges illustrated in Fig.3.7 are used to measure tangential, radial, or combined strains on thin membranes and diaphragms. Gauges of these types are available with diameters less than 5 mm (0,2 in).

750559

Fig.3.7a. Gauge for measuring tangential strain

750560

Fig.3.7b. Gauge for measuring radial strain

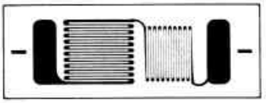
750561

Fig.3.7c. Gauge for measuring radial and/or tangential strain

Fig.3.7. Diaphragm strain gauges

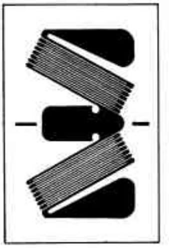
Stress Gauges

The stress gauges shown in Fig.3.8 can be calibrated to give readings directly in stress. They make use of the relationship demonstrated in Equation 2:4 that the stress in the measured direction is dependent upon the strain in that direction, and also upon the transverse strain.



3.8a

750562



3.8b

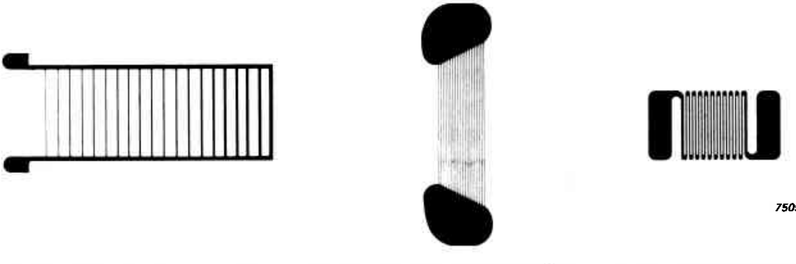
750564

Fig. 3.8. Typical stress gauge configurations

The gauge shown in Fig.3.8a has two grids with conductor lengths in the ratio of 1 : μ (Poissons ratio). The longer grid must be mounted with filaments parallel to the direction of the required stress measurement. When loaded, the two grids in series give the $\epsilon_x + \mu\epsilon_y$ term in the equation, so that the change in gauge resistance is proportional to stress. The other gauge in the figure uses a slightly different arrangement to achieve the same effect. When the central axis of the gauge is oriented in the direction of the desired measurement, the proportion of the gauge conductor in the direction of measurement and at right angles to it are in the ratio 1 : μ, so again the $\epsilon_x + \mu\epsilon_y$ term is given.

Gauges for Other Applications

Parameters other than stress or strain can be measured using strain gauge measuring instrumentation and suitable gauges. The examples shown in Fig.3.9 are fairly typical of non-strain measuring gauges. The



750563

3.9a. Crack propagation gauge

3.9b. Crack propagation gauge

3.9c. Temperature sensor

Fig. 3.9. Gauges for non-strain measurement applications

crack propagation gauges change in resistance as the growing crack progressively breaks the gauge conductors. Alternatively when a suitable gauge material is used that has a high sensitivity to temperature and very low sensitivity to strain, the resultant gauge makes an excellent temperature sensor. Typical sensors of the type illustrated have a useful temperature range from —80° to + 300°C (—100° to + 600°F).

The Strain Gauge as a Transducer

Resistance strain gauges are also used as the active element in several transducer applications, and the following short section briefly sketches some of the more typical examples. The significance of the bridge connection diagrams will be explained more fully in subsequent chapters on "The Measuring Circuit" and "The Practical Measuring System".

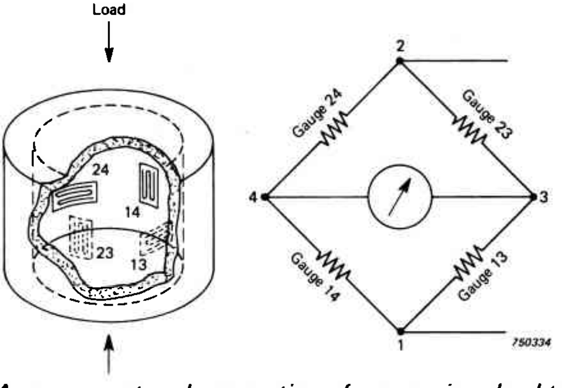


Fig.3.10. Arrangement and connection of gauges in a load transducer

The Load Transducer shown here makes use of four strain gauges cemented on the inside of the cylinder (for protection). The gauges are connected together to make use of Poissons ratio to increase the effective gauge factor, so that the bridge sensitivity will be approximately 2,6 times the sensitivity of an individual gauge, (see also Fig.6.7).

Under the action of a tensile load, the curvature of the Ring in Fig.3.11 is "flattened" so that the inner gauges experience tension while the outer gauges are in compression. The bridge sensitivity with this arrangement will be four times that of a single gauge, (see also Fig.6.8).

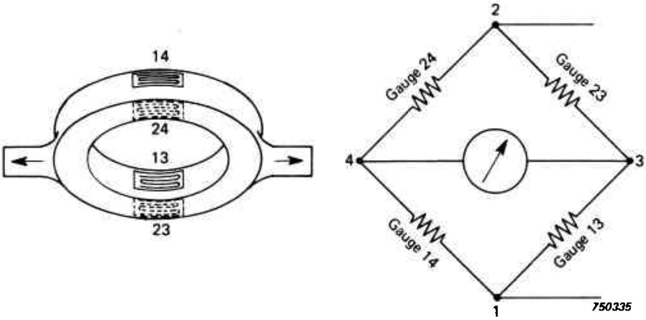


Fig.3.11. Ring Dynamometer for measuring tensile loads

The Pressure Transducer shown in Fig.3.12 has four strain gauges mounted on a thin circular diaphragm that bends under the action of pressure. The gauges at the edge of the diaphragm in a comparatively unstressed area give automatic temperature compensation as described in Chapter 6, while the gauges at the centre of the diaphragm are bent to give an output that is directly proportional to pressure.

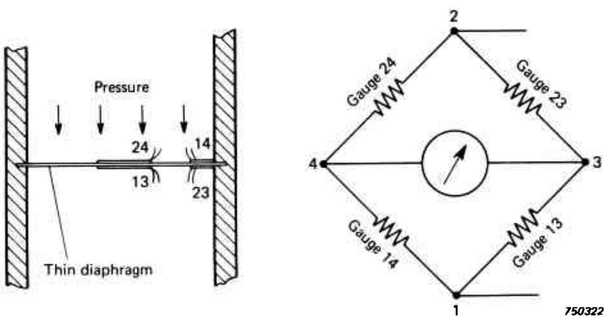


Fig.3.12. Diaphragm transducer for measuring pressure

The output from the strain gauges on the feeler arm shown in Fig.3.13 will be directly proportional to bending, which is proportional to displacement. The two gauges give automatic compensation for temperature changes at the gauges. They will have a combined sensitivity that is two times that of a single gauge.

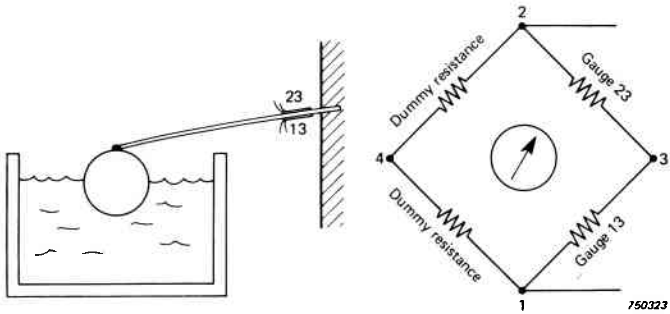


Fig. 3.13. Displacement Transducer for measuring liquid level

Backing Materials

Strain gauge backing materials have two main functions, they protect the gauge grid from damage during handling and mounting, and they transmit the strain from the test object to the gauge conductor.

The backing must have comparatively low stiffness so that it can follow strain changes in the test specimen without itself affecting them (by damping for example). On the other hand, it must be stiff enough to support the gauge conductor and not deform to give a distorted or irregular change in length. Simply stated, the backing must be stiff compared with the conductor, but flexible compared with the test material. As will be seen, this is not too difficult a requirement to meet, and several different types of backing material are in common use.

Paper backings are often employed with wire strain gauges. Paper has the twin advantages of being readily obtainable and easily worked, while its stiffness is adequate to support the gauge during handling. When properly applied with a thick layer of nitro-cellulose cement that completely penetrates all the pores in the paper, the backing is quite strong enough to transmit strains from the measuring object to the gauge conductor. This combination of cement and paper typically will only suffer a breakdown of adhesion at room temperature with strains greater than 100000$\mu\epsilon$, which is well above the limit of most conductor materials. This backing has a useful temperature range up to 70°C (150°F) for static strain measurements. Higher temperatures can be tolerated for short periods — up to a few hours, or for measuring dynamic strain.

Epoxy plastics are also used frequently as backing materials for many types of strain gauge, particularly for foil gauges. Epoxy plastic can operate successfully at slightly higher temperatures than paper, and it has a strain limit in excess of most conductor materials.

Applications for higher temperatures, up to about 200°C (400°F), can be satisfied by using phenolic resin (bakelite), or temperature cured epoxy plastic as a backing material. Bakelite backed strain gauges are also particularly recommended for their long term stability and resistance to creep. The higher temperature capabilities made possible by these backing materials mean that a careful choice of cement will be required. The properties of various cements and adhesives are described in the next chapter.

Stainless steel is also used as a carrier for gauges intended for operation at very high temperatures, up to 400°C (750°F). The conductor grid is bonded to the metal backing during manufacture, so that the difficult clamping and heat curing processes are carried out under accurately controlled conditions. Later the whole gauge assembly is simply mounted on the test specimen by spot welding.

For applications above 400°C, high temperature unbacked or strippable gauges are employed. These can be used typically up to temperatures of 900°C (1650°F). The unbacked conductor is cemented to the test specimen using a high temperature ceramic cement. It should be noted that specialized installation and curing techniques are required.

CHAPTER 4
STRAIN GAUGE ADHESIVES AND SEALING METHODS

Adhesives

As with the backing material, the major requirement for a strain gauge adhesive is sufficient elasticity to follow the dimensional changes in the test object without sacrificing accuracy in the transfer of strain to the gauge conductor. When selecting a suitable adhesive for cementing the strain gauge to the test object, several important factors have to be considered, as not all adhesives are usable for all gauges or for all measurement conditions. Some adhesives give a chemical bond, others make a mechanical connection, and still others a combination of the two.

Probably the most important factor to be considered will be compatibility with the gauge backing material and with the test object material. The cement must not damage either material, but make a sound and lasting bond with them. It should have good long term stability and not decompose or fall apart after a short time. Further, the cement should not be subject to appreciable creep during the lifetime of the test. Resistance to decomposition and creep must be maintained throughout the anticipated temperature range of the test, and be maintained up to the maximum strain level expected. Some tests demand stability up to very high temperatures and at high strain levels, and these requirements must be satisfied by the cement.

The cement should be a good electrical insulator eliminating the possibility of current leakage through the test specimen that could give rise to erroneous balance conditions. This is particularly important where stripped strain gauges are employed, especially if it is borne in mind that the resistance differences being measured can be of the order of thousandths of an ohm. After cementing the gauge in place, it is important that the insulation is checked using a good quality ohmmeter.

Some adhesives need special care and treatment during storage, strain gauge mounting, and curing. Some cements have to be stored at low temperatures to yield the best storage life and avoid wastage. Others need special curing processes, high temperatures for periods of several hours which the test object may not be able to tolerate. Cements based on refractory oxides require special equipment to handle the molten spray. Yet other cements present something of a health hazard as they release toxic fumes during cure, or require special precautions during handling to prevent harmful contact with the skin that causes inflamation.

Resistance to humidity is another very important quality for a strain gauge cement to possess. Some adhesives and carrier materials are far more responsive to changes in atmospheric humidity, and exhibit changes in insulation resistance that can give rise to zero drift and difficulties in balancing. Longer term, or outdoor strain gauge installations will require some form of moisture protection.

The chart shows the major types of adhesive used for mounting strain gauges, and lists some representative examples of manufacturers, together with the more important characteristics that influence selection of the most suitable cement for any particular measurement assignment.

A "Model" method for cementing the gauge

When cementing the gauges in place, cleanliness is of the utmost importance. The surface of the test specimen must be smooth enough for the cement type selected to adhere properly, and should also be free from rust, scale, paint, etc., that could interfere with the quality of the bond. Where necessary the surface can be machined to give a suitable finish, or glass paper and emery cloth can be used. Sometimes it may even be necessary to roughen a highly polished surface to ensure a satisfactory joint.

The prepared surface must be washed down thoroughly to remove metal or dirt particles and grease. Trichlorethylene, methyl-ethyl-ketone, or carbon tetrachloride are suitable cleaning fluids for this purpose. Ventilation precautions should be observed. Several washes will be necessary to achieve the necessary degree of cleanliness, and it is a good policy to keep washing with clean paper towels or lint free cloths until the wiping cloth no longer picks up dirt.

Survey of adhesives used for mounting strain gauges

Cement Type	Organic							Ceramic			
	Evaporation of solvent at room temp.	Meltable thermoplastic	Chemical Reaction					Evaporation hot curing	Chemical reaction hot curing	Molten spray	Very hot sinter
			Curing at room temperature			Hot curing					
Cement base	Nitro-Cellulose	Shellac	Acrylic	Polyester	Epoxy	Epoxy	Phenolic	Silicate	Phosphate	Refractory oxide	Glass
Representative example	Baldwin SR-4	DeKhotinsky	Eastman 910 Loctite IS-12B	Philips PR 9244	Araldite (various grades)	Araldite (various grades)	Bakelite Philips PR 9246	Baldwin RX-1	Brimor Cement	Rokide A Rokide C	Hommel L-6AC
Cure time (usually higher temp. gives accelerated cure)	2–24 h	Cooling time	5 min	5–15 min	12–72 h	2–10 h	1–6 h (room temp. cure takes 72 h)	1 h	1 h	None	30 min
Cure temperature	20–65 °C 70–150 °F	Melts at 130 °C (275 °F)	Room temp	Room temp	20–65 °C 70–150 °F	100–250 °C 210–500 °F	100–200 °C 210–400 °F	110 °C 220 °F	300–600 °C 600–1100 °F	None	1000 °C 1800 °F
Cure pressure	0.1–0.5 kp/cm^2 1,4–7 lb/in^2	0,1–1 kp/cm^2 1–15 lb/in^2	0,1–1 kp/cm^2 1–15 lb/in^2	0,5–1 kp/cm^2 7–15 lb/in^2	0,5–1 kp/cm^2 7–15 lb/in^2	0,5–3 kp/cm^2 7–50 lb/in^2	1–5 kp/cm^2 15–75 lb/in^2	None	None	None	None
Breaking strain of bond *	100000 $\mu\varepsilon$	20000 $\mu\varepsilon$	20000 $\mu\varepsilon$	20000 $\mu\varepsilon$	20000 $\mu\varepsilon$	20000 $\mu\varepsilon$	20000 $\mu\varepsilon$	5000 $\mu\varepsilon$	5000 $\mu\varepsilon$	20000 $\mu\varepsilon$	5000 $\mu\varepsilon$
Used with strain gauge types	Paper backed	All types that can take melt temperature	All except paper	All types	All types	All types that can take cure temperature	Only types with phenolic backing	Stripped, and ceramic insulated	Stripped and ceramic insulated	Stripped, and ceramic insulated	Stripped, and ceramic insulated
Electrical insulation	Excellent	Excellent	Excellent	Excellent	Excellent	Excellent	Excellent up to 150 °C (300 °F)	Decreases with higher temp.	Decreases with higher temp.	Excellent but decreases at higher temp.	Excellent
Moisture resistance	Fair	Good	Fair	Fair	Good	Good	Good	Poor, disolves in water	Fair	Good	Excellent
Remarks	Soluble in MEK and acetone, not suitable for plastics soluble in these	Glue stick melts on contact with heated test object	Has poor keeping qualities. Best for short term measurements	Has poor keeping qualities. Store at low temperatures	Two component cement, some hardeners are toxic	Two component cement, some hardeners are toxic	Has good keeping qualities.	Reacts adversely with some metals and plastics	Reacts adversely with some metals and plastics	Porous	Suitable for all metals that can take cure temp.

* At temperatures substantially below 0 °C the adhesive qualities of most organic cements are reduced

Cementing should take place immediately after the test surface has been cleaned, to prevent the collection of dust particles, or the formation of an oxide layer. The backing of the gauge should also be degreased immediately prior to cementing. Mark the position that the gauge is to occupy so that it can be accurately located while cementing, or lay the gauge in place and tape the supply leads so that it can be raised up for cementing without losing its location. It is often a good idea to tape a small piece of polythene foil so that it can cover and protect the gauge, but can also be hinged up.

When an adhesive is being used that requires mixing, it should be ready mixed and pre-curing — following the makers instructions — so that it can be applied immediately the gauge and surface are ready. Apply a thin layer of cement to the back of the gauge, or to the test specimen, and lay the gauge in place ensuring that it lies in the correct position and orientation. Press it down gently with a thumb to ensure that there are no air bubbles trapped between the specimen and the backing. Apply pressure and temperature during the curing period as required for the particular adhesive selected. If terminal tabs are to be cemented in place, this can also be done before curing.

After the cement has cured, the strain gauge leads can be soldered to the terminals. Whenever possible the gauge resistance should be checked for changes caused by damage during mounting. If it is found to be markedly different from the rated value, a new gauge will have to be cemented in its place. The gauge insulation from the test piece should also be checked using a megohmmeter that employs a low excitation voltage. Use of a high voltage can damage the gauge wire and possibly cause overheating and breakdown of the adhesive or backing. 50 to 100 megohms of insulation is considered to be adequate to ensure stable operation of the gauge.

The widely used method of running a finger tip over the mounted gauge to check whether it is firmly bonded is a useful test, provided the results are treated with caution. Any movement of the gauge, caused by faulty bonding should show up as a violent change in the indicated strain level on the measuring instrument.

If these tests of the gauge give a satisfactory result, the whole gauge arrangement can be given a water-proofing treatment if this is necessary for the particular test environment, or test program duration.

Sealing Methods

In the short term, it is permissible to use unprotected gauges under conditions of cleanliness and very low humidity. Indoor applications in the test or research laboratory can employ unprotected gauges for periods up to a few days without experiencing insulation changes or zero drift due to the penetration of moisture. However, **any** strain gauge installation that is intended for use over extended periods of time **must** have some form of protection to exclude moisture. Outdoor applications will require particularly stringent measures to counteract the effects of the weather. Various methods have been successfully used, varying from complete encapsulation in epoxy or phenolic compounds, to multilayer rubber and metal sandwich with a liberal use of sealing compounds.

Fig.4.1, shows one typical method recommended by several strain gauge suppliers whereby the whole gauge, and a short section of the supply leads are completely covered by a mastic compound. Most gauge manufacturers are able to supply a sealing kit for use with their gauges to protect them in this way.

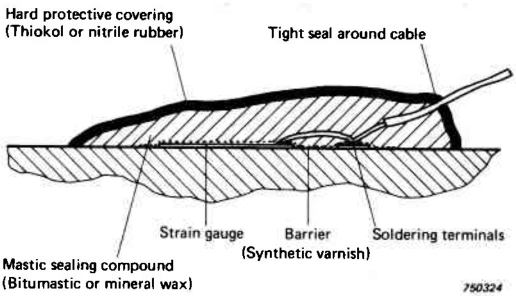


Fig.4.1. Typical strain gauge protection for longer term applications

The arrangement shown in **Fig.4.2** is recommended for really long term strain gauge installations and gives excellent protection against severe environments. The hard, metal or plastic cover gives protection against mechanical damage and also acts as a first stage barrier against the ingress of moisture. If a metal cover is used it should be made from as thin a sheet as possible to minimise its effect on the strain measured by the gauge. A small amount of silica gel ensures that any air that is trapped will be as dry as possible.

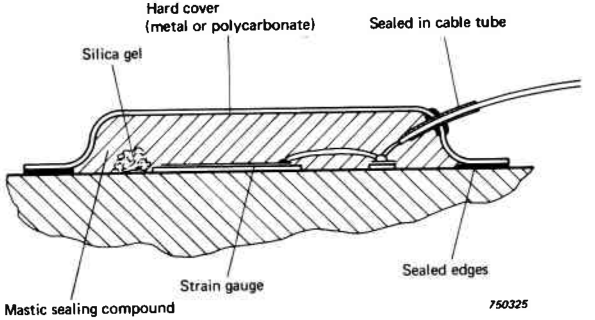


Fig. 4.2. Strain gauge protection for long term installations in severe environments

CHAPTER 5
SELECTION OF THE CORRECT STRAIN GAUGE

After discussing the merits and otherwise of strain gauges, backing materials and adhesives in the preceding chapters, the following section is a quick, common sense guide to the selection of the correct gauge for any particular application.

As the individual measuring problem will have a great influence on the actual gauge type and configuration chosen, the following points should be considered:

1. Straight forward linear strain distributions where the directions of the principal planes are known can be measured by the classic single element gauge.

2. Where some augmentation of the Gauge Factor is desirable, a double grid gauge, such as that shown in **Fig.3.5c** may be employed, or two separate gauges.

3. If the strain distribution is more complex, one of the various types of strain rosette may be more useful or more convenient.

4. The size of the gauge must be suitable for the test object. If high strain gradients are expected use the smallest possible gauge, When it is necessary to employ higher voltages for gauge excitation, a larger gauge grid may be required to dissipate the extra heat. 500 mW per square centimetre (5 W per in^2) is a fair approximation for the maximum amount of power that a typical foil gauge can handle when mounted on a metal test piece. This is the equivalent of 1 V on a 120 Ω gauge that occupies 0,17 cm^2 (0,025 in^2) of area, but it should be remembered that a wire gauge may be able to take only a quarter of this heating value.

5. The gauge, backing material, and cement chosen must be able to operate at the temperatures and strain levels anticipated. Where these levels are not known suitable safety factors should be applied.

6. The gauge, backing material, and cement should have a long enough fatigue life to survive the test program without premature breakdowns.

7. The backing material and cement must be chemically compatible with the test object material, and not produce unwanted side-effects.

8. The temperature expansion coefficient of the gauge should suit the material to which it is going to be attached. This item is rather less important when dummy gauges or full bridge connections are used for temperature compensation as described in Chapter 6.

9. Finally as this is a common sense guide, use the least expensive gauge that satisfies the conditions listed above.

Strain gauges and strain gauge transducers can be obtained from the sources listed in the following Table.

Strain Gauge Suppliers

Supplier	General Purpose Gauges	Semi-conductor Gauges	Trans-ducers	Special Purpose Gauges
Ailtech, USA				Concrete, High temp.
Akers Electronics, Norway			x	
Baldwin Lima Hamilton, USA	x	x	x	High extension High temp.
William T. Bean Inc., USA	x			High extension
Bell & Howell, USA			x	
Budd Company, USA	x		x	High temp.
Dentronics Inc., USA	x			High temp.
Ether Ltd., UK		x	x	
Frischen Electronic, German Fed. Rep.	x			
Fritz Hellige & Co., German Fed. Rep.	x			High temp.
Gauge Technique, UK	x		x	
Hottinger Baldwin Meßtechnik, German Fed. Rep.	x	x	x	High extension High temp.
Hitec Corp., USA	x			High temp.
Kulite Semiconductor Products, USA		x	x	
Kyowa, Japan		x		
Herbert Lembcke, Sweden	x			
Magnaflux, USA	x			High extension
Micro Gauge Inc., USA		x		
Micro Measurements, USA	x		x	High extension High temp.
Peekel, Holland				High extension
Philips, Holland	x	x		
Proud, Czechoslovakia	x			
Showa, Japan	x			
Strainstall, UK				Concrete High temp.
Statham Instruments Inc., USA			x	
Technograph, UK	x		x	
Transducers (CEL) Ltd., UK	x		x	
Vibro Meter Corp., Switzerland	x	x		High temp.
Vishay Intertechnology, USA	x		x	High extension High temp.
Welwyn Electric, UK	x		x	High extension High temp.
Whittaker, USA		x	x	

CHAPTER 6
THE MEASURING CIRCUIT

As mentioned in Chapter 1, the extremely small changes of the order of thousandths of an ohm, that occur in the gauge resitance (typically 120 Ω to 2000 Ω) due to variations in the applied strain can be measured by the Wheatstone bridge. The bridge is shown in Fig.6.1, where the resistance shown in each of the four bridge arms could represent a strain gauge. The measuring indicator can be a galvanometer, or the signal can be led to the input terminals of a measuring amplifier — as in the B & K Strain Indicator Type 1526.

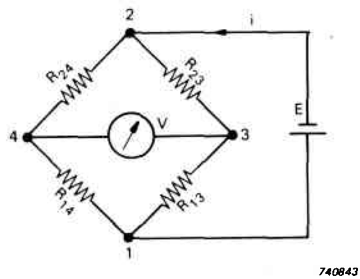


Fig.6.1. The Wheatstone Bridge

The numbering convention used here is the same as that adopted for the Type 1526 Instruction Book.

The symbols used in this section are as follows:

R = resistance (Ω)
δR = change in resistance (Ω)
i = current (amp)
V = voltage (volt)
E = excitation voltage (volt)

First consider the ideal case where the resistance in each arm is identical, i. e. the bridge is "balanced" so there is no potential difference across the indicator. Hence the voltage drop across R_{24} must be equal to the voltage drop across R_{23}, and a definite relationship must exist between the resistances and their positions in the bridge arms.

As $\quad V_{24} = i_{241}\ R_{24} = V_{23} = i_{231}\ R_{23}$

and $\quad V_{14} = i_{241}\ R_{14} = V_{13} = i_{231}\ R_{13}$

then $$i_{241}\ R_{24} = i_{231}\ R_{23} \tag{6:1}$$

and $$i_{241}\ R_{14} = i_{231}\ R_{13} \tag{6:2}$$

dividing 6:1 by 6:2

$$\frac{i_{241}\,R_{24}}{i_{241}\,R_{14}} = \frac{i_{231}\,R_{23}}{i_{231}\,R_{13}}$$

so $$\frac{R_{24}}{R_{14}} = \frac{R_{23}}{R_{13}} \tag{6:3}$$

This is a very important relationship which indicates that any change in the resistances on one side of the bridge can be balanced by adjustments to the resistance values on the other side of the bridge. As will be shown later, this is the basis for one of the methods used for temperature compensation. Further, it indicates the possibility of increaseing the effective gauge factor when several gauges are used in the bridge arms.

Solving for one bridge resistor (for example R_{24}),

$$R_{24} = \frac{R_{23}\,R_{14}}{R_{13}} \tag{6:4}$$

Equation 6:4 indicates that when the value of R_{14} is known accurately, and the ratio of the two resistances on the other side of the bridge is also known accurately, the value of the unknown R_{24} can be determined with the same degree of accuracy.

Taking the example a stage further, if the unknown resistance R_{24} was a strain gauge on a test piece, the bridge could be balanced accurately before loading, the loading could be applied to the specimen, and then the bridge could be balanced again by adjusting the ratio of the two resistors R_{23}/R_{13}. If the change in this ratio is known, then the difference in strain gauge resistance between the two loading conditions can be determined. This difference will be due to the strain caused by the applied load, and the strain can be calculated from,

$$\text{Strain} = \frac{\delta R}{Rk} = \frac{\delta R}{R} \times \frac{1}{k} \tag{3:1}$$

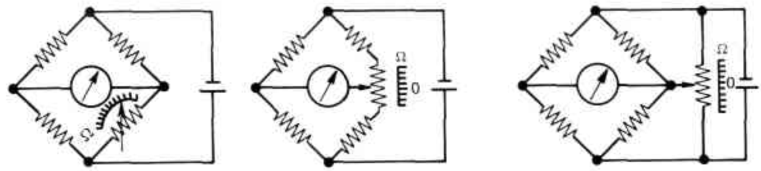


Fig.6.2. Methods of balancing the Wheatstone Bridge

The ratio of the two resistors can be changed by an accurately determined amount when one or more of the bridge arms contains a calibrated potentiometer. Fig.6.2 shows some of the typical arrangements that are used in commercial instruments. A variation of the arrangement shown in Fig.6.2c is used in the B & K Type 1526 to give a convenient method of balancing the bridge.

As shown in equation 3:1 above, the resistance change is directly proportional to the strain, so the variable resistor control can easily be calibrated to read strain, thereby eliminating some tedious arithmatic. An even more convenient arrangement is achieved when the indicator itself is calibrated directly in strain values. If the bridge is balanced at the start of each test, the strain level can be read from the meter without the need for re-balancing the bridge for each load condition. This arrangement works very well with small strain levels, but at higher levels as the bridge goes further out of balance the galvanometer begins to load the bridge introducing distortion.

When the indicating instrument is a measuring amplifier, the loading effect can be kept to a minimum because modern amplifiers can be built to work with very low input levels. If a digital voltmeter is used as the display it will be just as easy and economical to obtain fine resolution in voltage measurement as it would be in resistance. Furthermore, the gain can be adjusted to suit the different gauge factors encountered when gauges of various materials are used, thus preserving the direct reading feature.

The Quarter Bridge

What effect will a resistance change due to strain have on the voltage measured by the indicator? Referring once again to the bridge shown in Fig.6.1, let a change in resistance be introduced into one of the arms to unbalance the bridge, so that R_{24} becomes R + δR, while the other resistances remain unchanged (= R). This is equivalent to the situation where only one (active) gauge is being subjected to strain, while the other resistances (known as dummies) can be either inactive gauges, or just resistors, this arrangement is the "Quarter Bridge".

$$i_{241} = \frac{E}{R_{24} + R_{14}} = \frac{E}{R + \delta R + R}$$

$$= \frac{E}{2R + \delta R}$$

$$V_{24} = i_{241}\, R_{24}$$

$$= \frac{E}{2R + \delta R}(R + \delta R)$$

$$= \frac{ER + E\delta R}{2R + \delta R} \qquad (6{:}5)$$

while
$$i_{231} = \frac{E}{R_{23} + R_{13}} = \frac{E}{2R}$$

$$V_{23} = i_{231}\, R_{23} \quad = \frac{ER}{2R}$$

$$= \frac{E}{2} \qquad (6{:}6)$$

So the voltage in the measuring link,

$$V = V_{23} - V_{24} = \frac{E}{2} - \frac{ER + E\delta R}{2R + \delta R}$$

$$= \frac{2ER + E\delta R - 2ER - 2E\delta R}{4R + 2\delta R}$$

$$= \pm \frac{E\delta R}{4R + 2\delta R}$$

For very small values of δR,

$$4R + 2\delta R \doteqdot 4R$$

So that
$$V = \pm \frac{E\delta R}{4R} \qquad (6{:}7)$$

Substituting the values from equation 3:1, and rearranging,

$$\varepsilon = \frac{4V}{Ek} \quad (strain) \qquad (6{:}8)$$

This is a very important relationship showing that there is a direct connection between the applied strain and the voltage in the measuring link. If the excitation voltage and the gauge factor are known, it is only necessary to measure the out-of-balance voltage to obtain the strain level.

The strain given by equation 6:8 could have been produced equally easily by mechanical loading, or by the effects of a temperature change if the strain gauge had not been matched to the expansion coefficient of the test specimen material. It could also have been caused by a combination of load and temperature changes, but there is no simple way of telling which. Fortunately the Wheatstone bridge principle allows for the insertion of a compensation resistor into one of the bridge arms. This resistor has a temperature characteristic calculated to compensate for the resistance change in the active gauge due to a temperature variation.

The simplest and most commonly employed method to obtain a compensation resistor with the desired characteristic, is to use a strain gauge with identical specification to the active gauge. This compensation gauge is mounted on an unstressed part of the test specimen, or on a separate piece of the same material placed close to the test piece so that it experiences the same temperature changes without the strain changes. The compensation gauge must be connected into one of the bridge arms adjacent to the active gauge arm, (positions R_{14} or R_{23} in Fig.6.1). Many measuring arrangements use calibrated resistors in positions R_{23} and R_{13}, so the compensation resistor is most conveniently positioned at R_{14}.

Referring again to Fig.6.1, let R_{24} and R_{14} both increase by an amount δR_1 due to a temperature change.

$$i_{241} = \frac{E}{R_{24} + R_{14}} = \frac{E}{2(R + \delta R_1)}$$

$$V_{24} = i_{241}\ R_{24}$$

$$= \frac{E}{2(R + \delta R_1)}(R + \delta R_1)$$

$$= \frac{E}{2} \qquad (6:9)$$

and as V_{23} is unchanged

$$V = V_{23} - V_{24}$$

$$= \frac{E}{2} - \frac{E}{2} \qquad = 0 \qquad (6:10)$$

So the bridge is still balanced, and the temperature effect is compensated. If the active strain gauge had been subjected to a strain and a temperature change, while the compensation gauge was only subjected to a temperature change, the temperature effects would have been cancelled out. Only the strain component would have appeared as a voltage in the measuring link, which is illustrated as follows.

Let R_{24} increase by an amount δR_1 due to a temperature change and an amount δR due to an applied load, while R_{14} increases by δR_1 only, due to the temperature change.

$$i_{241} = \frac{E}{R_{24} + R_{14}} = \frac{E}{R + \delta R_1 + \delta R + R + \delta R_1}$$
$$= \frac{E}{2R + 2\delta R_1 + \delta R}$$

$$V_{24} = i_{241}\ R_{24}$$
$$= \frac{E}{2R + 2\delta R_1 + \delta R}(R + \delta R_1 + \delta R)$$
$$= \frac{ER + E\delta R_1 + E\delta R}{2R + 2\delta R_1 + \delta R} \qquad (6{:}11)$$

From 6:6 $$V_{23} = \frac{E}{2}$$

And $$V = V_{23} - V_{24}$$
$$= \frac{E}{2} - \frac{ER + E\delta R_1 + E\delta R}{2R + 2\delta R_1 + \delta R}$$
$$= \frac{2ER + 2E\delta R_1 + E\delta R - 2ER - 2E\delta R_1 - 2E\delta R}{4R + 4\delta R_1 + 2\delta R}$$
$$= \pm\frac{E\delta R}{4R + 4\delta R_1 + 2\delta R}$$

For very small values of δR_1 and of δR

$$4R + 4\delta R_1 + 2\delta R \doteqdot 4R$$

so that $$V = \pm\frac{E\delta R}{4R}$$

The temperature components have cancelled each other out, and only the strain component due to loading remains, i.e. this is equation 6:7 again.

The Half Bridge

Up until this point, only a bridge with one active gauge has been considered. However equation 6:3 indicated the possibility that if more gauges are positioned on the test specimen in such a way that their resistances change under the action of an applied load, and these gauges are then connected into the measuring bridge, several advantages can be achieved. Small strain levels will be detected more easily because **the additional active gauges will give a greater out-of-balance signal** when the bridge is loaded, temperature influences can be cancelled out as shown in the preceding example, and in many arrangements greater measuring accuracy will also be obtained. The following examples illustrate some of the possibilities.

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Fig. 6.3. Test specimen in simple bending with Half Bridge connection

Fig. 6.3 shows a test specimen with applied loading that puts one surface into compression while the other surface is in a numerically equal tension. When the connections in the bridge are as shown in the diagram (known as "Half Bridge", R_{24} will increase to R + δR under the action of the bending while R_{14} will decrease to R — δR if identical strain gauges are used.

$$i_{241} = \frac{E}{R_{24} + R_{14}}$$

$$= \frac{E}{R + \delta R + R - \delta R} = \frac{E}{2R}$$

$$V_{24} = i_{241}\ R_{24} = \frac{E}{2R}\ (R + \delta R)$$

$$= \frac{ER + E\delta R}{2R}$$

Unchanged from 6:6 $V_{23} = \dfrac{E}{2}$

so
$$V = \frac{E}{2} - \frac{ER + E\delta R}{2R}$$
$$= \frac{ER - ER - E\delta R}{2R}$$
$$= \pm \frac{E\delta R}{2R} \qquad (6{:}12)$$

When this result is compared with equation 6:7 it will be seen that the bridge sensitivity has been doubled.

Provided the test specimen has its neutral plane at the centre of the test strip, the bending stresses can be calculated from the surface strains found by this measuring method.

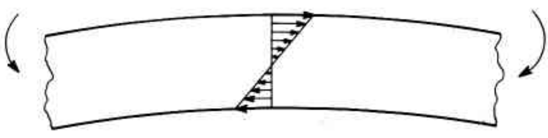


Fig. 6.4. Stress distribution on a test strip in simple bending

If in addition to the pure bending, the specimen had also been subjected to a tensile force, both gauges would also have been stretched by the same amount, in addition to the changes caused by the bending. The stress distribution for this situation is shown in Fig. 6.5.

As the two active strain gauges are connected in adjacent arms of the measuring bridge, the effect of the tension load will be cancelled out, leaving just the bending components to be measured.

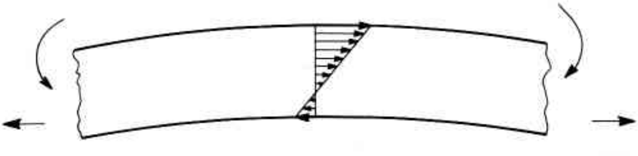


Fig. 6.5. Stress distribution on a test strip in combined bending and tension

Furthermore, if strain gauges without temperature compensation, or gauges with poorly matched compensation had been used, an apparent strain will be detected by both gauges if the temperature varies. Again, as the gauges have identical temperature characteristics, and are connected in adjacent arms of the bridge, the temperature effects will cancel each other out.

The arrangement is automatically made independent of temperature variations, the only condition being that the two gauges really must be experiencing the same temperature. Note that with large specimens, or where temperatures vary greatly, this condition might not be met, and self-compensating gauges must be used.

Fig.6.6 shows another test specimen with a tensile load applied. When the gauge connections are as indicated, R_{24} increases to $R + \delta R$, while R_{14} decreases to $R - 0{,}3\,\delta R$, because the test material in this case has a Poissons Ratio of 0,3.

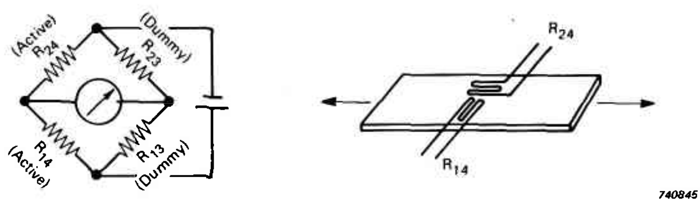


Fig.6.6. Specimen in pure tension with Half Bridge connection

$$i_{241} = \frac{E}{R_{24} + R_{14}}$$

$$= \frac{E}{R + \delta R + R - 0{,}3\delta R}$$

$$= \frac{E}{2R + 0{,}7\delta R}$$

$$V_{24} = i_{241}\ R_{24}$$

$$= \frac{E}{2R + 0{,}7\delta R}\ (R + \delta R)$$

$$= \frac{ER + E\delta R}{2R + 0{,}7\delta R}$$

Again $$V_{23} = \frac{E}{2}$$

And $$V = \frac{E}{2} - \frac{ER + E\delta R}{2R + 0{,}7\delta R}$$

$$= \frac{2ER+0,7E\delta R - 2ER - 2E\delta R}{4R+1,4\delta R}$$

$$= \pm \frac{1,3E\delta R}{4R+1,4\delta R}$$

And for very small values of δR

$$4R+1,4\delta R \doteqdot 4R$$

so that $$V = \pm \frac{1,3E\delta R}{4R} \qquad (6:13)$$

Again, comparing the result with equation 6:7, the bridge sensitivity will be seen to be 1,3 times the sensitivity for a single active gauge. The main use for this type of connection (other than the minor increase in sensitivity it affords) is to obtain automatic temperature compensation. It will be readily understood that if temperature changes occur during the test, both gauges will experience the same alteration of resistance so that the temperature effects are cancelled out as in the previous example. Note that if the compensation gauge is mounted on a strained part of the test sample, the bridge sensitivity will be affected.

The Full Bridge

The foregoing examples have illustrated that it is possible to improve the sensitivity obtainable from a single gauge, while maintaining temperature compensation, by suitable connection of an extra loaded gauge into the bridge. An even greater increase in sensitivity can be obtained when all four bridge arms contain active strain gauges. In this arrangement, called the "Full Bridge", balance must be obtained by some additional resistor in the bridge that can be adjusted, for example like the arrangements shown in Fig.6.2b or Fig.6.2c. When all of the active gauges are subject to the same temperature variations, the apparent strains due to temperature cancel out as before.

In the following example, a tensile test specimen has four identical active strain gauges mounted upon it with their connections as shown in Fig.6.7. If the Poissons Ratio of the test material is 0,3 again, the following changes in gauge resistance will occur under the action of the applied load,

R_{24} increases to $R + \delta R$
R_{13} increases to $R + \delta R$
R_{14} decreases to $R - 0,3\,\delta R$
R_{23} decreases to $R - 0,3\,\delta R$

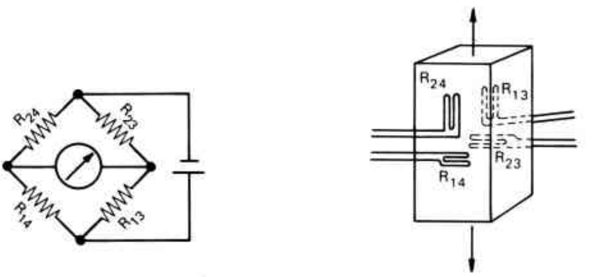


Fig. 6.7. Specimen in tension with Full Bridge connection

$$i_{241} = \frac{E}{R_{24} + R_{14}}$$
$$= \frac{E}{R + \delta R + R - 0{,}3\delta R}$$
$$= \frac{E}{2R + 0{,}7\delta R}$$
$$V_{24} = i_{241} \ R_{24}$$
$$= \frac{E}{2R + 0{,}7\delta R} \ (R + \delta R)$$
$$= \frac{ER + E\delta R}{2R + 0{,}7\delta R}$$

When the full bridge is used V_{23} is still the product of i_{231} and R_{23}, but the result is no longer E/2 as this side of the bridge now contains active gauges.

$$i_{231} = \frac{E}{R_{23} + R_{13}}$$
$$= \frac{E}{R - 0{,}3\delta R + R + \delta R}$$
$$= \frac{E}{2R + 0{,}7\delta R}$$
$$V_{23} = i_{231} \ R_{23}$$
$$= \frac{E}{2R + 0{,}7\delta R} \ (R - 0{,}3\delta R)$$
$$= \frac{ER - 0{,}3E\delta R}{2R + 0{,}7\delta R}$$

And $$V = V_{23} - V_{24}$$

$$= \frac{ER - 0{,}3E\delta R}{2R + 0{,}7\delta R} - \frac{ER + E\delta R}{2R + 0{,}7\delta R}$$

$$= \frac{ER - 0{,}3E\delta R - ER - E\delta R}{2R + 0{,}7\delta R}$$

$$= \pm \frac{1{,}3\delta R}{2R + 0{,}7\delta R}$$

For very small values of δR,

$$2R + 0{,}7\delta R \doteqdot 2R$$

So that $$V = \pm \frac{1{,}3\delta R}{2R} \qquad (6{:}14)$$

If this result is compared with equation 6:7 again, it will be seen that **the full bridge connection on a tensile (or compressive) specimen has** had the effect of making the bridge sensitivity 2,6 times the sensitivity of a single active gauge.

Fig.6.8 shows another specimen in bending, with a full bridge strain gauge arrangement. When the gauges are connected as illustrated, the following changes in resistance occur under the action of the loading,

R_{24} increases to R + δR
R_{13} increases to R + δR
R_{14} decreases to R — δR
R_{23} decreases to R — δR

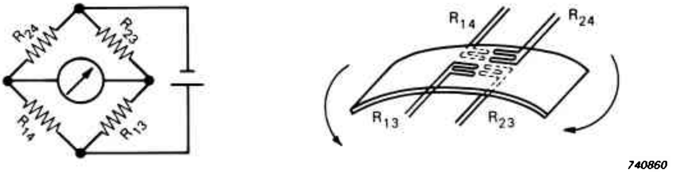


Fig.6.8. Specimen in Bending with Full Bridge connection

$$i_{241} = \frac{E}{R_{24} + R_{14}}$$

$$= \frac{E}{R + \delta R + R - \delta R} = \frac{E}{2R}$$

And $$V_{24} = i_{241}\ R_{24}$$
$$= \frac{E}{2R}\ (R + \delta R)$$
$$= \frac{ER + E\delta R}{2R}$$

Similarly $$i_{231} = \frac{E}{R - \delta R + R + \delta R} = \frac{E}{2R}$$

And $$V_{23} = i_{132}\ R_{23}$$
$$= \frac{ER - E\delta R}{2R}$$

Therefore the voltage in the measuring link,

$$V = V_{23} - V_{24}$$
$$= \frac{ER - E\delta R}{2R} - \frac{ER + E\delta R}{2R}$$
$$= \pm \frac{E\delta R}{R} \qquad (6{:}15)$$

In the bending case with a full active bridge, it is possible to increase the effective bridge sensitivity to four times the sensitivity of a quarter bridge with only one active gauge (6:7). Note that this is only possible with a bending condition where two gauges can be placed to measure positive strain, and the other two placed to measure an equal negative strain.

The foregoing examples are not intended to be an exhaustive list of all possible methods of connecting measuring bridges, or of balancing them. However it is hoped that the basic principles established here will enable the following chapters on practical measuring systems, with their limitations and possible sources of error to be understood. Further, that other practical arrangements will suggest themselves to the test engineer faced with the problem of strain measurement.

CHAPTER 7
THE PRACTICAL MEASURING SYSTEM

The B & K Strain Indicator Type 1526 (shown in Fig.7.1) is a direct indicating instrument with a digital display of the measured strain level. Several features help to make this instrument particularly simple to set-up and use. The 1526 has facilities for measuring with full, half, and quarter bridge connections, and its sensitivity range is from ± 199,9 $\mu\epsilon$ to ± 19990 $\mu\epsilon$ with one active strain gauge. It can employ gauges with resistances between 50 Ω and 2000 Ω, and there is also provision for varying the gauge factor to suit different gauges. The Strain Indicator is able to measure both static and dynamic strain at frequencies up to 300 Hz, and there is a hold function to capture transients. A 3 kHz squarewave carrier system is employed, with bridge voltages adjustable to 3 V, 1 V, and 0,3 V. Up to five bridges can be connected to the instrument at the same time, and then be selected one at a time for measurement.

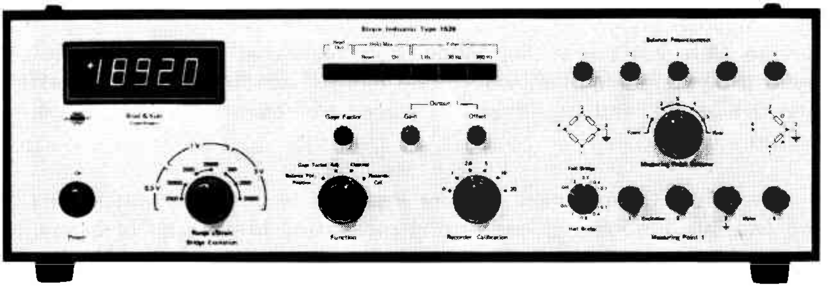


Fig. 7.1. The B & K Strain Indicator Type 1526

The relative significance of the various features will now be discussed in more detail.

Digital displays are simple to use, completely eliminating the needle parallax that can give balancing or reading inaccuracies. The display on this instrument is particularly good having 0,05% resolution and high accuracy, with all the illuminated elements in the same plane to give easy reading at wide viewing angles. The digital signal is also available in BCD form from a TTL compatible output if it should be needed for further digital processing.

This instrument is also intended to be used with B & K Level Recorders Type 2305 and Type 2307, therefore known DC levels can be shown on the display while a matching calibrated DC output is available for setting up the Recorder.

Built-in circuitry allows the user to select use with full or half bridge measurement, and a reminder on the front panel indicates the gauge connections to be used.

Fig.7.2 shows the bridge connections used for full and half bridge arrangements. It will be seen that when half bridge is selected, a pair of precision 1 kΩ resistors make up the dummy half of the bridge, and that the two active gauges are connected in positions R_{23} and R_{13}. It will also be seen that the balance resistor is placed in parallel with the active gauges. The other resistors in the balance line enable the balance resolution to be varied. The values shown on the Mode Selector switch refer to the balance resolution obtainable. With 120 Ω gauges and a gauge factor of 2,00, position "2%" allows 120 Ω/6,04 kΩ = 2% to be balanced, equivalent to ± 10000 $\mu\epsilon$ which gives a resolution of 2000 $\mu\epsilon$ per turn of the potentiometer. A better resolution and finer balance can be achieved (at the cost of a reduction in the total balancing range available) when the Selector is switched to a lower percentage value.

When quarter bridge connection is required, the Strain Indicator is switched to half bridge mode and a special Quarter Bridge Adaptor con-

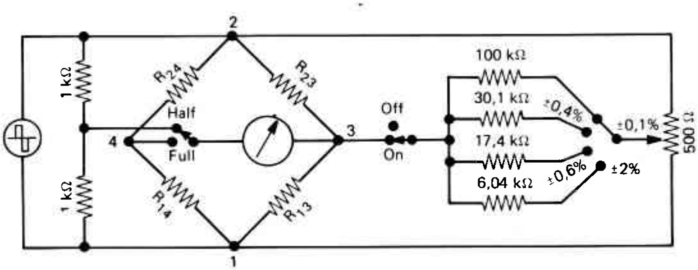


Fig.7.2. Bridge and balance connections used in the Type 1526

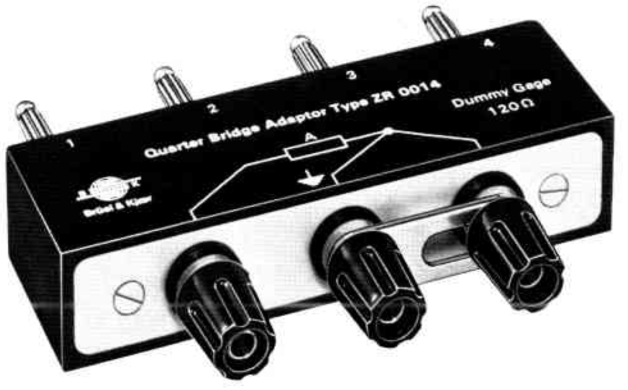


Fig. 7.3. The Quarter Bridge Adaptor ZR 0014

taining a precision resistor as a dummy gauge is plugged into the instrument. Fig. 7.3 is a photograph of the Adaptor. The pins on the rear engage with the terminals on the 1526 while the connections are shown on the top surface. The photograph shows a strap joining two of the terminals, but this introduces a resistance error into the bridge and is not recommended for really accurate measurements.

In the recommended arrangement for connecting the one active strain gauge to the Quarter Bridge Adaptor (Fig. 7.4) the conductors R_L should be as similar to each other as possible. This can easily be achieved by using the same type of conductor lead, and keeping the lead lengths the same. The two leads are connected as shown, and a third lead (R) connects one of the terminal tabs on the strain gauge, via the Adaptor, directly to Terminal 3 on the 1526 so that each arm in the half bridge contains 120 Ω plus the lead resistance R_L. This ar-

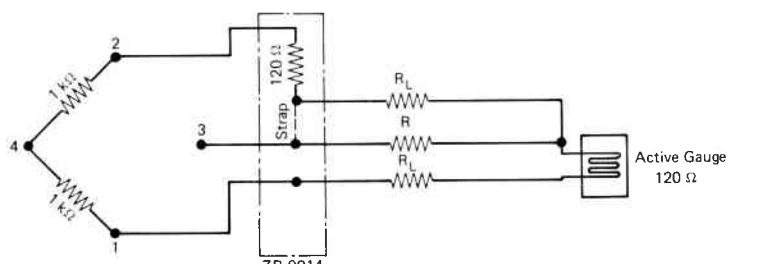


Fig. 7.4. Connections recommended for the Quarter Bridge Adaptor

rangement ensures that any resistance change in the leads due to temperature variations will be equally divided between the two bridge arms, thus maintaining the balance. The effect of the additional lead in the measuring link is negligible because resistance R is very small compared with the input impedance of the measuring amplifier, but there will be a very small reduction in the effective k as described in Chapter 8.

The connection R may be replaced by the strap — shown with a broken line, but the use of this strap is recommended only where connection leads are short, and where there are no temperature variations. This is because when the strap is used the bridge arm containing the active gauge also contains two lead resistances, while the other arm will only contain the 120 Ω dummy. The latter situation can cause erroneous measurements because, although the bridge can be balanced, all the temperature induced resistance changes in the leads will now occur in only one arm of the bridge. This problem of temperature effects on the leads is discussed further in Chapter 8.

The Quarter Bridge Adaptor ZR 0018 is available for use with 350 Ω strain gauges. The construction and use of the ZR 0018 is in all respects identical to the ZR 0014, except for the resistance value of the dummy gauge.

Note that when a quarter bridge arrangement is being used, a temperature compensated strain gauge must be employed which has a temperature characteristic matched to the test material as was described in Chapter 3.

As was demonstrated in Chapter 6, the bridge sensitivity must be considered for each individual measuring arrangement because both the gauge factor of the gauges used, and the bridge connections will affect it. The examples given show that the bridge sensitivity can take very different values, and it should also be born in mind that the engineer making the measurements may be more interested in obtaining a stress value, or the level of the loading that causes the strain. These parameters will require an extra multiplication factor to be applied to the bridge sensitivity. It is advantageous to have an instrument where the gauge factor can be adjusted over a full decade so that the levels displayed can have the correct numerical value for the parameter required.

The gauge factor, bridge sensitivity, or any other multiplication factor can be adjusted by means of the "Gauge Factor" control on the 1526.

Fig. 7.5. The Calibration Bridge ZR 0013

The range of adjustment is between approximately 0,90 and 10,70 with an accuracy of ± 0,01%. An accuracy of ± 0,005% can be obtained when the Calibration Bridge ZR 0013 is used. The gauge factor can be shown on the display for adjustment while a measuring arrangement is being set-up.

The Calibration Bridge contains a finely balanced set of resistors that have an ohmic difference corresponding to a 2000 $\mu\epsilon$ display value with a gauge factor of 2,00.

In some circumstances DC excitation can cause a thermocouple effect because of temperature differences between soldered joints, or at other places in the arrangement where dissimilar conductors are in contact. To avoid these effects, the Type 1526 employs an AC carrier wave system. AC levels are easier to amplify, and an additional advantage of the carrier system is that it permits low bridge excitation voltages, which in turn minimise heat generation. On the 1526 the minimum excitation voltage is 0,3 V, and when this is used with 120 Ω gauges, it gives a total of only 75 mW per gauge to be dissipated as heat, which permits the instrument to be used for measurements on materials with very low thermal conductivities. When a higher excitation voltage is used, the signal to noise ratio in the amplifiers is improved, so that much lower strain levels can be resolved. With the highest excitation level available on this instrument (3,0 V) it is possible to measure up to plus or minus 200 $\mu\epsilon$ full scale, with a resolution of 0,1 $\mu\epsilon$.

A major disadvantage of AC excitation is the sensitivity of the measuring system to reactive unbalance, usually in the form of capacitive unbalance caused by long bridge leads. Various methods have been devised to counteract capacitive imbalance, the most common being to include variable capacitors in the bridge arms and then to achieve a capacitive and a resistive balance. The Type 1526 makes the extra stage — capacitive balancing — unnecessary by using a 3 kHz square wave as the excitation voltage, and by gating only a portion of the measuring signal from the bridge for measurement as shown in Fig.7.6. This gating effectively eliminates the signal peaks caused by the capacitive unbalance in the bridge. With the demodulator principle used, the residual unbalance is sensed as resistive unbalance, which can be balanced out with the resistance balance potentiometer. Typically a 0,05 μF imbalance on a 120 Ω bridge will cause an error in reading of ± 1 digit in the 1 V: 2000 $\mu\epsilon$ range.

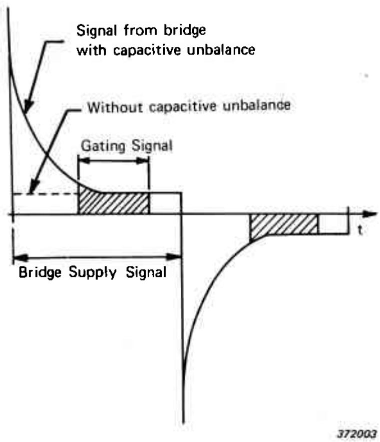


Fig. 7.6. Operation of the gating signal

When measuring static strain, the measured value can be read directly from the digital display, together with an indication of whether the strain is tensile or compressive, shown by a + or a — sign. The sign always follows the strain being experienced by the gauges in positions R_{13} and R_{24}.

The Type 1526 can also be used to measure dynamic strain, but now the reading procedure is slightly different because the dynamic levels cause the displayed values to fluctuate unreadably and the positive and negative signs to alternate. This situation is immediately stopped by pushing in the "Hold Max." ON button to bring the peak hold function into operation. This captures the maximum positive or negative strain level, whichever was the greater, and a steady display results.

If both static and dynamic strain levels are present, as in the examples shown, it is still possible to use the features built in to the instrument to separate them. One of the low pass Filters which suppress frequencies above 3 Hz, 30 Hz, or 300 Hz can be used to selectively suppress the dynamic component of the strain so that just the static level is displayed.

Alternatively, the method detailed in the following example can be followed. Push the Hold ON button as for a dynamic signal to yield a positive or negative maximum level as before. This level, which is shown as "A" in Fig.7.7, should be noted. The Function Selector should be switched to BALANCE POTENTIOMETER POSITION and this displayed value noted. Select OPERATE mode again and adjust the Balance Potentiometer carefully, depressing the Hold RESET button from time to time to keep a check on the display until it just changes sign. Note the new display level — this is value "B" in the figure. The static strain level can be found by subtracting B from A, and the dynamic level by multiplying B by 2. The lesser maximum can be found by subtracting 2 X B from A. Examination of the figure shows that the changed balance setting has in effect moved the base line by an amount equivalent to the static level, so that the dynamic levels fluctuates equally on either side of it.

Should it be necessary to restore the Balancing Potentiometer to its original position for any reason, this can be done quite easily without unloading the specimen. BALANCE POTENTIOMETER POSITION is se-

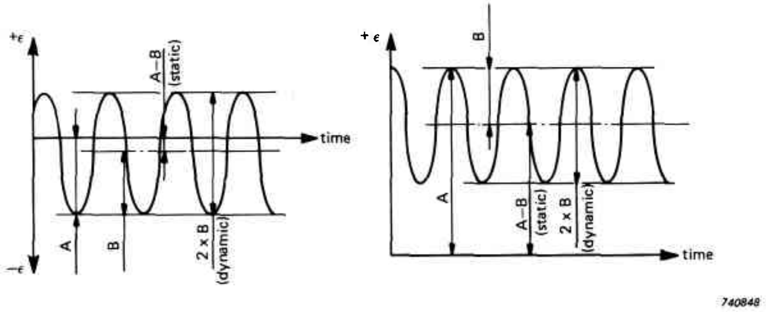


Fig.7.7. Examples of combined static and dynamic strains

lected again and the Potentiometer adjusted until the displayed value is the same as the value originally recorded for the Balance Potentiometer Position. The Potentiometer is now on exactly the same setting as it was when value "A" was measured, and this is also the same as the original zero balance from the commencement of the test.

Another method of determining the dynamic component of the strain is to use an AC voltmeter (for example a Type 2425, or Type 2426), or an oscilloscope to measure the AC component on the DC level coming from one of the DC outputs.

It is often required that measurements from several different points on the test specimen be obtained, and it is very convenient when this can be done without re-connecting and re-balancing measuring bridges during the course of the test. Up to five bridges can be connected to the 1526 at the same time, in any combination of quarter-, half-, or full-bridges, and the bridges can be selected one after another for measurement.

When more than five points have to be measured, the Type 1544 Multipoint Selector and Control enables the 1526 to be connected in turn to a series of measuring points automatically, so that the strain level can be measured. Up to ten measuring points can be connected to the 1544, and if even more points are required, Type 1545 Multipoint Selectors with provision for ten more points in each unit can be connected to the same Selector and Control 1544 to give a maximum of four hundred measuring points. Selection of each point is controlled by the 1544 which also has a digital display to indicate the identification number of the measuring point currently being sampled.

Each measuring point connection on both 1544 and 1545 has two terminals for the excitation signal which is derived from the 1526, and two terminals to receive the measuring signal. Each point connection has its own "Bridge Mode" switch and "Balance" potentiometer.

Automatic sweep rates from ten measuring points per second to one point every ten seconds can be selected. Point selection can also be accomplished remotely by a TTL source, or by the cam switch of a Level Recorder Type 2305 or Type 2307. When necessary, individual measuring points can be switched out so that the point sweep passes over them in the automatic sequence, whole decades — individual 1545 units — can also be by-passed in this way. One single measuring point can be selected for measurement by a "Call" switch, and if several

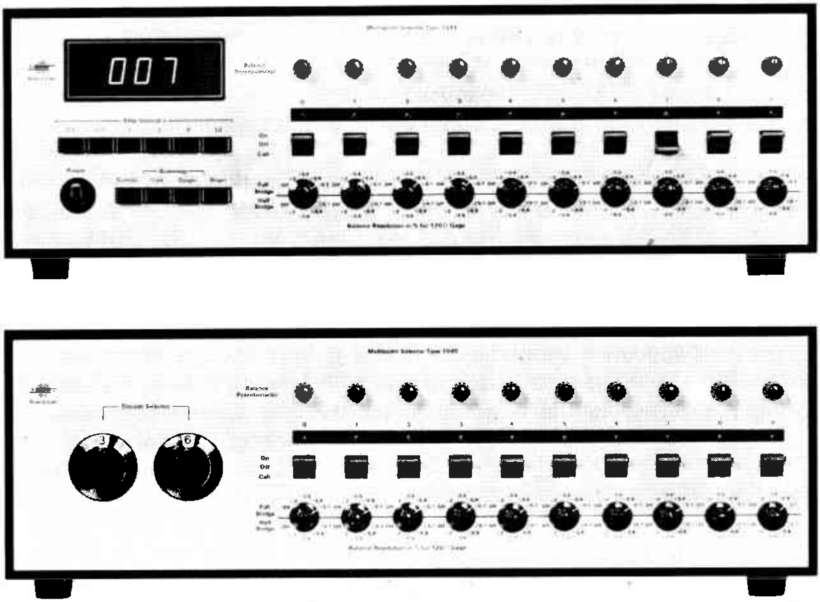


Fig. 7.8. The Multipoint Selector and Control Type 1544 and the Multipoint Selector Type 1545

points are "called" in this manner, a restricted sweep sequence over just these points will occur.

The built-in interface in the Selector and Control 1544 is a B & K Standard Interface. It is similar to the proposed IEC Standard, the exception being that it is not addressable. The data transmitted from the 1544 consists of the measuring point identity number, and the strain level.

The instruments that have been described in this chapter make up a very flexible measuring arrangement that is simple to operate, and capable of performing a wide range of measuring assignments.

CHAPTER 8

SOURCES OF ERROR IN STRAIN GAUGE MEASUREMENTS

Many different factors can cause measuring errors in strain gauge systems. These sources fall into three broad categories, electrical errors that arise in the measuring system, errors that have their source in temperature induced effects, and errors due to faulty gauge mounting or selection.

This chapter will analyse some of the more common sources of error and suggest ways to improve measuring accuracy by proper selection of system components, and by careful preparation of each item. It will also show how some measuring errors can be compensated for automatically, and how the effects of others can be minimised by suitable calibration procedures.

Electrical Sources of Error

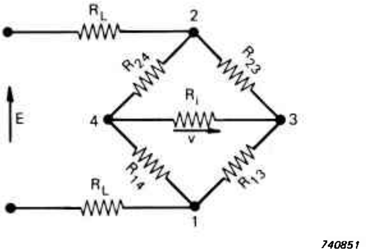


Fig.8.1. Full-bridge showing some electrical sources of error

The figure shows the familiar bridge circuit, with the addition of lead resistances R_L and input resistance to the measuring instrument R_i. A general expression for the amplification can be written as follows,

$$\frac{V}{E} = \frac{R_{23}\ R_{14}}{(R_{23}+R_{13})\,(R_{24}+R_{14})}\left[\frac{\delta R_{24}}{R_{24}} - \frac{\delta R_{23}}{R_{23}} + \frac{\delta R_{13}}{R_{13}} - \frac{\delta R_{14}}{R_{14}}\right](1-T)NP \qquad (8{:}1)$$

If the bridge is in balance to begin with.

$$\frac{R_{24}}{R_{14}} = \frac{R_{23}}{R_{13}} = a \tag{8:2}$$

And if

$$\frac{\delta R}{R} = r \tag{8:3}$$

Then the amplification expression simplifies to,

$$\frac{V}{E} = \frac{a}{(1+a)^2}\ (r_{24} - r_{23} + r_{13} - r_{14})\ (1 - T)\ NP \tag{8:4}$$

Where the terms (1 — T), N, and P are error factors from various sources as will be explained later.

Supply leads

The leads that supply the measuring bridge are by far the most serious potential source of errors in any strain gauge arrangement. The leads should be suitable for the particular test application, for example it is pointless to waste time and money on thoroughly waterproofing the gauge installation where poor quality leads allow the penetration of moisture.

Chapter 7 mentioned methods for minimising errors caused by temperature changes on long leads, but wherever possible, long leads should be excluded from the actual bridge arms. Where this is not possible, the supply leads should still be as short as practicable, and made up from the same length and type of cable to minimise resistive and capacitive effects. The lead resistance should be kept as low as possible to exert the smallest influence on the overall resistance of the bridge system.

To get the lead resistance problem into proportion, consider the following example as illustrated in Fig.8.2, with a single active 120 Ω gauge having k = 2,00.

The leads will act as an integral part of the gauge resistance so that any variation in the ambient temperature influences the indicator reading directly. Heavy gauge leads with a comparatively low resistance will be required, but even so the lead influence can be considerable with

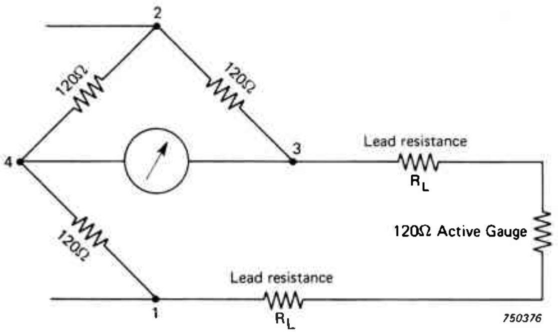


Fig. 8.2. Quarter bridge arrangement with all lead resistance in the same arm as the active strain gauge

this arrangement. If the leads are 10 metres (33 ft) in length with a diameter of 0,8 mm (0,03 in = 21 gauge), and are made from copper with a resistance of 0,017 Ω for a 1 mm² conductor 1 metre in length.

$$R_L = \frac{0{,}017 \times 10}{0{,}5} = 0{,}34\ \Omega$$

So that the total additional resistance in the bridge arm is 0,68 Ω.

Copper has a temperature coefficient of 0,004/°C (0,0022/°F), so that a 1°C change in temperature will change the total lead resistance by

$$\delta R_L = 0{,}68 \times 0{,}004$$

$$= 0{,}0027\ \Omega/°C$$

This value can be substituted for the δR term in equation 3:1, to yield an apparent strain component due to temperature effects on the supply leads.

Apparent strain $= \dfrac{\delta R}{Rk}$

$$= \frac{0{,}0027}{120 \times 2{,}00}$$

$$= 0{,}000011$$

$$= 11\ \mu\varepsilon/°C\ \ (6\ \mu\varepsilon/°F)$$

To eliminate this source of error, the single active gauge should be connected into the measuring bridge with the three lead system that

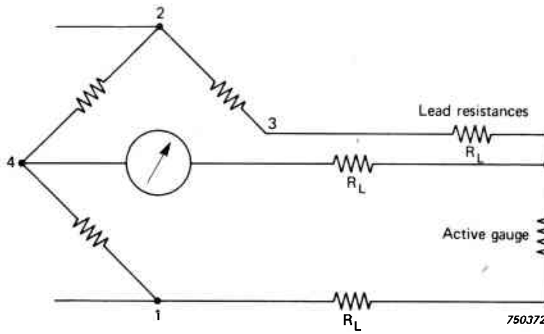


Fig.8.3. Recommended method of connecting single active gauge with three lead system

is shown in Fig.8.3 the guiding principle being to ensure that any changes in resistance due to temperature effects on the leads will be divided between two adjacent arms of the bridge cancelling each other out. To further ensure that the leads really experience the same temperatures, it is quite common to twist them together.

Having now (hopefully) eliminated this source of error due to temperature effects on the supply leads, there still remains the basic modification to the bridge resistance caused by lead resistances.

Error due to lead resistance can be calculated as follows.

Error Factor $$P = \frac{1}{1 + \frac{R_{cu}}{R_B}} \qquad (8:5)$$

Where R_{cu} is the total resistance in the supply leads and R_B is the resistance of the bridge that must be overcome by the supply (i. e. the re-

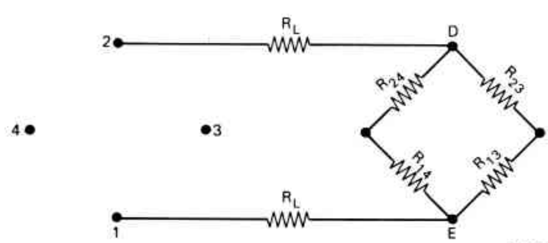


Fig.8.4. Full-bridge with lead resistances shown

sistance measured between points D and E in Fig.8.4 when the bridge is disconnected from the measuring instrument.

Relative error $$1 - P = \frac{1}{1 + \dfrac{R_B}{R_{cu}}} \qquad (8:6)$$

As an example, consider a full-bridge with four 120 Ω gauges connected to the measuring instrument by leads that each have a resistance R_L = 0,23 Ω.

$$1 - P = \frac{1 \times 100}{1 + \dfrac{2 \times 120/2}{2 \times 0,23}} = 0,382\,\%$$

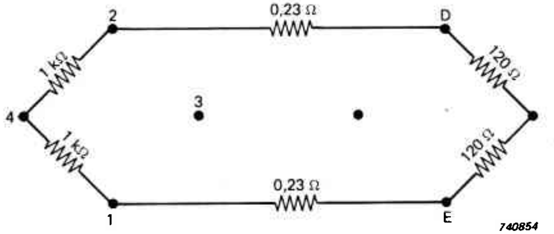


Fig.8.5. Half-bridge with lead resistances shown

For the half-bridge shown in Fig.8.5, connected to a Type 1526 having precision 1 kΩ resistors, and the same gauges and lead resistances as before,

$$1 - P = \frac{1 \times 100}{1 + \dfrac{2 \times 120}{2 \times 0,23}} = 0,192\,\%$$

A quarter bridge with the same resistance values, and connected as recommended in Fig.8.3 will have the same relative error as the half-bridge example.

Pick-up in the supply leads can also be a problem, but it is greatly reduced by twisting the leads around each other as for eliminating temperature error, or even better using a cable with a properly grounded screen. When the connections have been made, it is best to allow the leads to be moved as little as possible to avoid transients, especially after the bridge has been balanced.

Bridge non-linearity

When a direct reading instrument is being used, the output from the measuring bridge will not always be an exact linear function of the change in gauge resistance, because the bridge is out of balance when the measurement is made. The (1 — T) term in equations 8:1 and 8:4 is the error factor due to measuring strain with the bridge unbalanced.

The relative error
$$T=\frac{\dfrac{(r_{23}-r_{13})\,(r_{23}-ar_{13})}{1+r_{23}+a(1+r_{13})}-\dfrac{(r_{24}-r_{14})\,(r_{24}-ar_{14})}{1+r_{24}+a(1+r_{14})}}{r_{24}-r_{23}+r_{13}-r_{14}} \qquad (8:7)$$

When a = 1, which will be the case when the bridge is made up from a set of matched strain gauges, or matched gauges and precision resistors, the expression can be simplified to,

$$T=\frac{r_{24}^{2}-r_{23}^{2}+r_{13}^{2}-r_{14}^{2}}{2(r_{24}-r_{23}+r_{13}-r_{14})} \qquad (8:8)$$

If R_{24} and R_{14} are used as active gauges, the expression further simplifies to,

$$T=\frac{1}{2}\,(r_{24}+r_{14}) \qquad (8:9)$$

When R_{24} and R_{14} are active gauges with the opposite sign and the same magnitude, like those shown in Fig.6.3, the relative error T will be 0.

If only one active gauge is in use, and the balance circuit is also in use, it is best to connect the gauge into the opposite side of the bridge from the balance circuits whenever possible. When the 1526 is being used with a quarter bridge adaptor, the influence of the balance circuit on R_{13} should be taken into consideration. Using equation 8:7 with *a* = 1 and only R_{24} = 0,

$$T=\frac{r}{2} \qquad (8:10)$$

A rule of thumb which can be used for the case with one active strain gauge says that the relative error factor T is the same size as the measured strain (in strain) when gauge factor 2 is used,

as $\qquad \varepsilon k=r$

then $\qquad T=\frac{r}{2}=\frac{\varepsilon k}{2} \qquad (8:11)$

For example: with gauge factor 2, the indicated strain for a system with one active gauge is 1713 $\mu\epsilon$

Therefore $T = strain = 0{,}001713$

And $1713 \times 0{,}001713 = 2{,}93\ \mu\varepsilon$

So the corrected measurement should be 1716 $\mu\epsilon$.

Note that this can give large errors when semiconductor gauges with high gauge factors are being used.

Use of the balance circuit

When a balancing circuit is being used, the balance potentiometer alters the resistance value of one or more arms of the bridge. If these resistances happen to be strain gauges, their sensitivity to strain will be reduced, thereby introducing an error. This effect can be shown by means of an example.

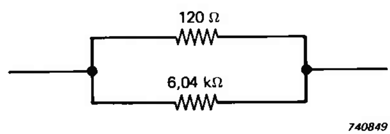


Fig.8.6. The components of R_{13}

Consider the 1526 used with one active 120 Ω gauge in position R_{13} and the full 500 Ω range of the potentiometer used to balance it when the "2%" resolution mode has been selected, (this is the worst case with a 120 Ω gauge). In effect, resistance R_{13} will be made up from two resistors as shown in Fig.8.6.

$$\frac{1}{R_{13}} = \frac{1}{120} + \frac{1}{6040}$$

$$R_{13} = 117{,}662\ \Omega$$

So the error in measurement will be,

$$= \left(\frac{120 - 117{,}662}{120}\right) 100\,\%$$

$$= 1{,}948\,\%$$

Similar calculations can be made for the errors that occur with other gauge resistance values, and with other balancing methods

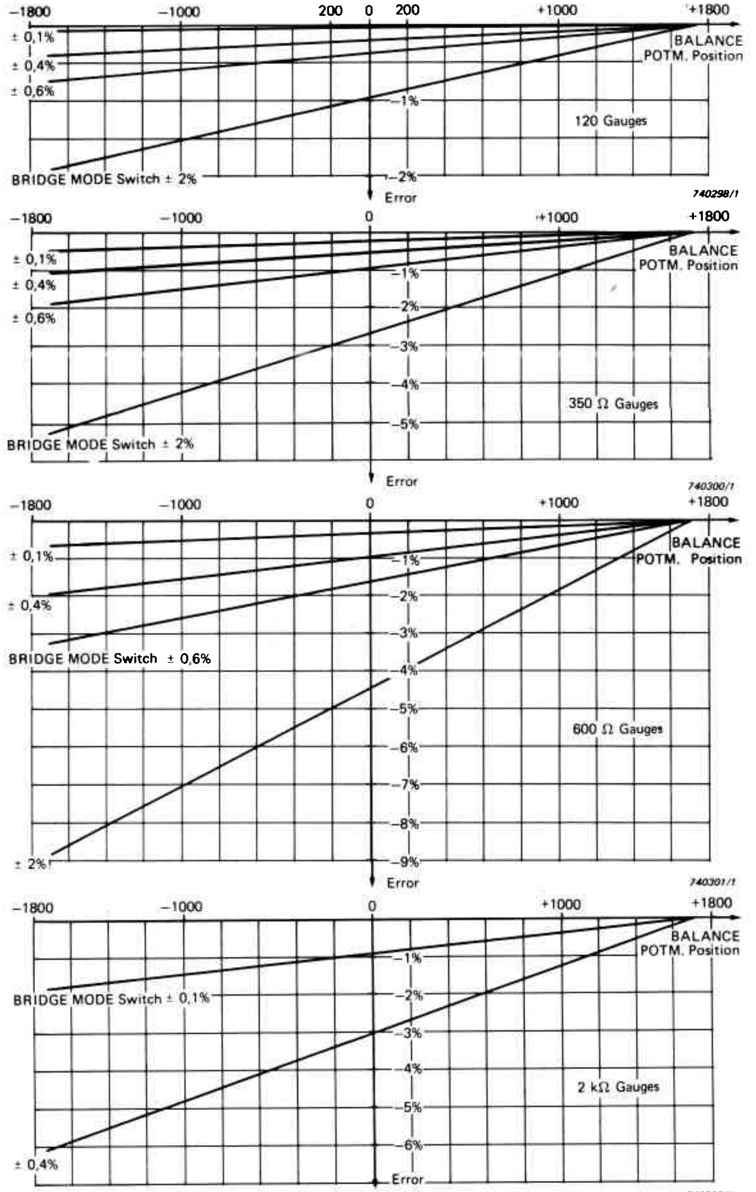


Fig. 8.7. Error due to use of balance circuit, with gauge resistances as shown, and a single active gauge used in position R_{13}, (with R_{23} the curves will slope the other way)

Error due to input impedance of the measuring instrument

The error factor due to the input impedance,

$$N = \frac{1}{1+\frac{R_o}{R_i}} \qquad (8{:}12)$$

where R_0 is the resistance of the bridge between points 4 and 3 with the measuring instrument disconnected, and R_i is the input impedance of the measuring instrument, (see Fig.8.8).

The relative error $$1 - N = \frac{1}{1+\frac{R_i}{R_o}} \qquad (8{:}13)$$

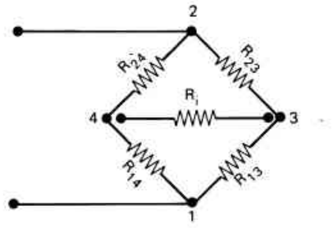


Fig.8.8. Input impedance of measuring instrument

Using the 1526 which has an input impedance of 67,5 kΩ as an example, and a full bridge with four 120 Ω strain gauges, the bridge resistance measured between points 4 and 3 will be,

$$\frac{1}{R_o} = \frac{1}{120+120} + \frac{1}{120+120}$$

So $$R_o = 120\ \Omega$$

$$1 - N = \frac{1}{1+\frac{67500}{120}}$$

$$= 1{,}78 \times 10^{-3} \quad = 0{,}178\,\%$$

Some measuring instruments are calibrated for use with strain gauges having one particular resistance value. The 1526 is calibrated in such a way that when it is used with a full bridge employing four 120 Ω strain gauges the error just calculated above will be compen-

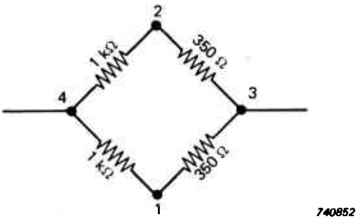


Fig.8.9. Half-bridge with 350 gauges

sated automatically. However, this compensated error will have to be deducted from the total error when a calibrated instrument is used with a bridge arrangement having different resistance values. This can be illustrated by means of an example. Fig.8.9 shows a half-bridge arrangement using two 1 kΩ resistors and two 350 Ω strain gauges.

The bridge resistance between points 4 and 3,

$$\frac{1}{R_o} = \frac{1}{1000+350} + \frac{1}{1000+350}$$

$$R_o = 675\ \Omega$$

With the 1526 $$1 - N = \frac{1}{1+\frac{67500}{675}}$$

$$= 9{,}90 \times 10^{-3} \quad = 0{,}990\,\%$$

The measuring error with respect to the calibration with 120 Ω full bridge,

$$= 0{,}990 - 0{,}178$$

$$= 0{,}812\,\%$$

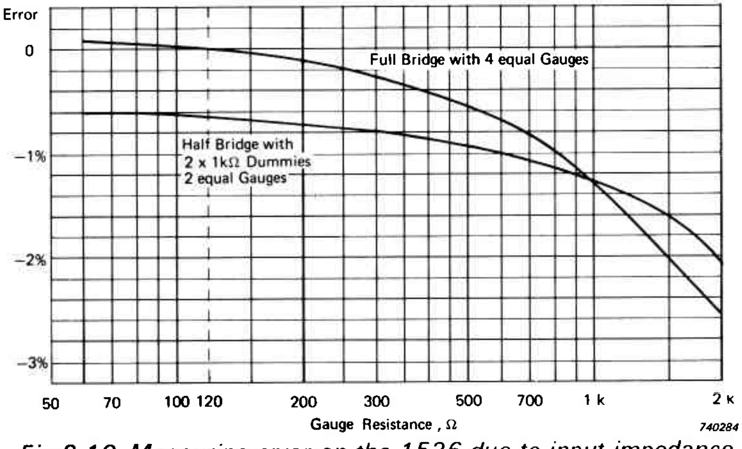


Fig.8.10. Measuring error on the 1526 due to input impedance

A chart can be calculated for the errors arising from different combinations of gauge resistance in full and half-bridges for any instrument. Fig.8.10 is a typical graph showing this type of measuring error.

Temperature variations

Changes in the temperature of the measuring system can cause errors for several reasons. First, the resistance, and hence the k factor of the gauge conductor material change with temperature to cause an apparent strain. Another possible cause of error is that the coefficient of expansion with temperature for the gauge may not be the same as that of the test object's material. A third reason is that the resistance of the supply leads changes with temperature, this was discussed in the previous section.

Various methods can be used to eliminate, or compensate temperature induced measurement errors. Use of dummy gauges as described in Chapter 6, will compensate for variations in the k factor with temperature, and for differences in the coefficient of expansion between the gauge conductor material and the test object material. The dummy is cemented in the same way as the active gauge to a piece of similar material to the test object, and connected in the measuring bridge in an arm adjacent to the active gauge. Being exposed to exactly the same temperature variations as the active gauge, the resistance changes due to temperature will be the same as in the active gauge, so their effects will be cancelled out. Compensation with this method is suitable for arrangements having one, two, or four active gauges.

When the dummy arm of a quarter bridge is made up from a calibrated resistor, or when it is impossible to subject the dummy to precisely the same temperature variations as the active gauge, one of the following alternatives can be applied. Self-compensating strain gauges have been developed that are immune to temperature variations. A calibrated length of another conductor material, or a thermocouple is included in the measuring grid, and has the opposite temperature/resistance characteristic to the grid material so that errors due to variations in the gauge resistance with temperature will be cancelled out.

Furthermore, manufacturers have gone to great lengths to develop conductor materials that exhibit only a very small resistance change over quite extensive temperature ranges when they are cemented on to the correct test specimen material. Manufacturers can supply gauges with temperature expansion coefficients to suit the most commonly used materials, and this type of gauge can often be employed when the temperature variations are not expected to exceed perhaps 50 to 80°C. Even when the temperature range is greater, the error in measurement is usually less than 2 $\mu\epsilon$ per °C.

Gauge mounting and connection

Incorrect mounting of the gauges on the test specimen can produce major errors due to creep, moisture effects, poor electrical insulation from the test piece, etc., as mentioned elsewhere. Gauge alignment is also very important in eliminating errors, and when possible, a preliminary test run using an application of brittle lacquer can give valuable information on the direction of strain "flow" to assist in the correct positioning of gauges.

The effects of moisture changing the balance point by altering the gauge insulation were mentioned in the chapter on adhesives. The best safeguard is a thorough water-proofing program to prevent the penetration of moisture into the cement or backing.

Diffusion is a long term effect, where atoms actually migrate from cement to the backing or to the test material and cause changes in insulation, or even a breakdown of the cement bond. The use of inert materials can help greatly to alleviate this effect.

After the first load cycle has been applied to the test specimen, and then removed again, it is sometimes found that the balance position has moved. This effect can have been caused by either of two conditions. If the load cycle has taken place in a comparatively short period of time, the change in balance point is very often due to hysteresis, as the other possible cause, creep, may take a longer period to show itself. To minimise the effects of hysterisis on the measurements, it is recommended that two or three load cycles are applied to the test object — if this is possible — before commencing the test. The balance point should be checked after each cycle, as after the second cycle the balance point will probably be stable within 0,1 or 0,2%, so that measurement can begin. After extended periods with alternating loading, an effect similar to hysterisis can occur again due to gauge fatigue.

All metals experience the phenomenon fatigue when they have been subjected to alternating stresses for longer periods of time. Strain gauge conductor materials are no exception to this rule, and this should be remembered when measurements are to be made on a system with alternating loads, particularly those operating at high frequencies. The total permitted number of strain reversals can be reached very rapidly. For example, taking a typical value for wire or foil gauges, 10^7 strain reversals at $\pm 1000\mu\epsilon$ can be achieved in less than three hours at 1000 Hz, or in just over a normal working day at 300 Hz, irrespective of whether or not measurements are being made. Fatigue failure of a gauge usually takes the form of a fracture where the conductor leads join the gauge grid, however, sometimes the adhesive breaks-down first.

Creep is a much more complex problem to diagnose or cure, because its effects, which are variable, rarely appear from the outset of the test. When a strain gauge has been subjected to a high strain level or has been placed in a high temperature environment, or some other environment capable of breaking down the bonding, the gauge may "creep" on the surface to which it was attached, or the gauge conductor may "creep" in the backing. Either of these effects, can cause substantial zero drift, or static strain readings that fall slowly with time as the elasticity of the gauge conductor material tries to pull the grid back to its unloaded configuration. Generally, alternating loads are less likely to give creep problems than are static loading systems.

Where only a few gauges are showing signs of creep, the most satisfactory remedy is to cement new gauges in place of the old, and simply discard the old gauge as attempts to remount a faulty gauge will almost always be fruitless.

In the situation where many gauges in a large installation are showing signs of balance point shift, or drifting strain levels, once the obvious possibility of wrongly applied loading has been eliminated and creep definately identified, a reappraisal of the gauging method is obviously called for. The adhesives and backing materials used must be completely suitable for the materials being tested. They must survive the anticipated temperature range, and the maximum strain level expected. Furthermore, the utmost care must be taken during the actual bonding of the gauge to the specimen, so that all mating surfaces are thoroughly prepared, and the adhesive correctly mixed.

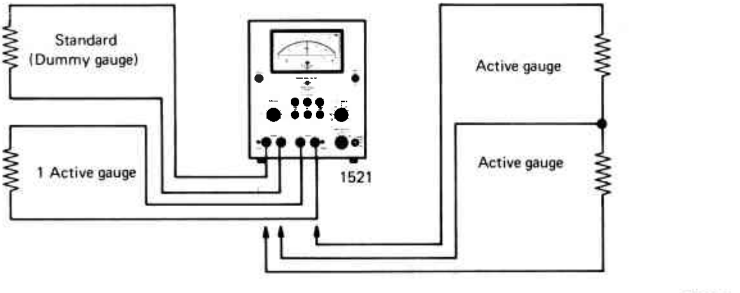
One important point, that is always worth remembering, a point valid for any type of strain gauge system, is that no matter how accurate the measuring instrument, or how well reasoned the choice of gauges and bridge arrangements may be, if the person who actually mounts the gauges on the test specimen made a mistake, the measurements can be misleading, or useless.

CHAPTER 9
MEASUREMENT OF STRAIN WITH OTHER B & K INSTRUMENTS

The engineer who has to measure strain only at very infrequent intervals, or who has to perform a preliminary survey to find whether strain measurement is of value to a particular test program, may be reluctant to invest in a Type 1526 Strain Indicator just for these "once in a while" measurements. This chapter will explain how useful strain measurements can be obtained from alternative arrangements based on B & K instruments that the user may already have in the test laboratory or inspection department.

Deviation Bridge Type 1521

A simple method of measuring static strains uses an instrument that may be found in the production testing, or receiving inspection department. Fig.9.1 shows the arrangement for measuring strain with a Deviation Bridge Type 1521. The 1521 compares the value of a resistance, inductance, or capacitance with a standard value, and gives a meter in-



9.1a. Measurement with one active gauge

9.1b. Measurement with two active gauges

Fig.9.1. Measurement of static strain using a Deviation Bridge Type 1521

dication of the percentage deviation from the standard. The instrument contains a measuring bridge circuit where the Standard and the Unknown form two arms of the bridge. When used in the Resistance Mode, a dummy strain gauge can be employed as the Standard resistor, and an active gauge as the Unknown. This arrangement is shown in Fig.9.1a.

Very briefly, setting up the instrument for measurement with this arrangement is as follows. The dummy gauge must have the same resistance as the active gauge, it is cemented to similar material to the test piece and mounted near it to give automatic temperature compensation. The dummy is connected to terminals 1 and 2 (Standard). The active gauge is connected to terminals 3 and 4 (Unknown).

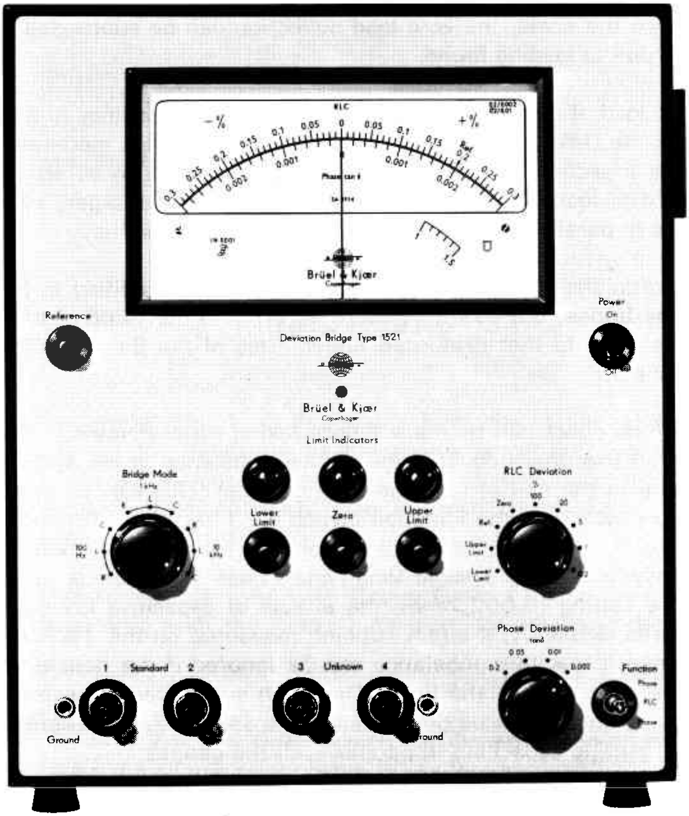


Fig.9.2. The Deviation Bridge Type 1521

Select Bridge Mode "R" with 100 Hz excitation frequency, and set the Function switch to RLC. Switch the RLC Deviation knob to "Zero" and adjust the zero position of the needle. Switch to "Ref." and adjust the reference level of the meter to "1" on the auxilliary scale. This sets the excitation voltage across each gauge to approximately 0,36 V.

Using the most sensitive scale (SA 0114, shown in Fig.9.2) and the most sensitive measuring range (0,2% RLC Deviation), it should be possible to read the zero load out-of-balance on the meter as a positive or negative percentage deviation. The tolerance on the gauges should be good enough to permit this, but if the deviation is larger than the 0,3% full scale deflection on the most sensitive scale, the Standard resistance can be adjusted to balance the unknown. Note that it is not necessary to balance the bridge so that the needle rests at the "0" point, because as long as the needle deflection for the loaded condition can be read from the scale, the zero load deflection can be subtracted and the change due to loading found.

At no load, if the needle goes off the scale at the positive end (indicating that the Unknown is the larger), a small resistance increment — for example a section of resistance wire — can be put in series with the Standard to increase its value. If the Standard is the larger, a large resistance in parallel with the Standard can reduce its value.

Note that the Unknown resistance can also be modified to help balance the bridge, but this introduces an error to the value of the Gauge Factor similar to that discussed under "Use of the Balance Circuit" in Chapter 8.

One final check can be made before taking measurements, when it is suspected that there could be capacitive unbalance in the system, (perhaps due to the use of long measuring leads). This can be determined by simply switching the Function switch to "Phase" and finding the deflection — if any. From a practical point of view, if the meter needle falls anywhere on the scale when the Phase Deviation is at its most sensitive setting (0,002 tanδ), the effects of capacitive unbalance can be ignored. Similarly, if strain is being measured on the 1% RLC Deviation range, capacitive unbalance can be ignored if the needle stays on the scale when 0,01 tanδ Phase Deviation is selected. If necessary, capacitive balancing can be performed in the same way as resistive balancing, by putting capacitors in parallel with the gauges.

Measurement will be explained by means of an example. A dummy and an active gauge, both with nominal resistance 120 Ω are connected in the arrangement shown in Fig.9.1a. The gauges have a Gauge Factor k = 2,00 and in the unloaded condition the instrument indicates + 0,08% deviation.

When the specimen is loaded, the needle indicates —0,24% deviation.

Referring back to Equation 3:1,

$$k = \frac{\delta R}{R} / \frac{\delta l}{l}$$

This can be rewritten as $$\varepsilon = \frac{\delta R}{kR} \qquad (9{:}1)$$

Where the δR term can be determined from the instrument deflection.

In the example, the total deflection due to loading

$$\delta R = -0{,}24 - (+0{,}08)\,\%$$

$$= -0{,}32\,\%$$ i. e. the active gauge is in compression)

Substituting into 9:1 $$\varepsilon = \frac{-0{,}32 \times 120}{2{,}00 \times 120 \times 100}$$

$$\varepsilon = -0{,}0016\,\varepsilon \quad = -1600\,\mu\varepsilon$$

Two points are obvious from this result. First, the actual value of the resistance is not too important as the terms cancel out. Second, the loading caused a change in reading of 32 graduations of the scale, which is equivalent to 1600 $\mu\epsilon$. That is, with Gauge Factor 2, one scale graduation represents 50 $\mu\epsilon$.

The instrument can be used with Gauge Factors that do not give an "easy" relationship with the meter indication, by adjusting the Reference level.

For example, if the Gauge Factor for a particular gauge is 2,2 set the Reference level to 2/2,2 = 0,91 (which can be estimated on the main scale), to retain the simple relationship 50 $\mu\epsilon$/graduation.

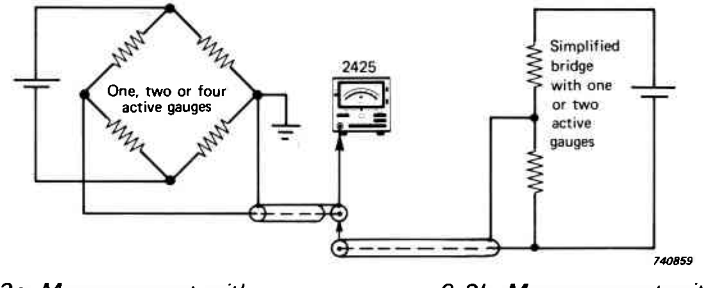
When more sensitivity is required, two active gauges can be employed, one in the Standard arm and one in the Unknown arm, as shown in Fig.9.1b. Making use of the Poisson effect will give approximately 40$\mu\epsilon$/graduation with Gauge Factor 2, and if the test piece is in bending with one active gauge in tension and one in compression 25$\mu\epsilon$/graduation can be achieved. Note that only three leads are required with two active gauges as terminals 2 and 3 are connected internally.

Dynamic strains can also be measured using a 1521 as this instrument features a DC output that can be fed to an oscilloscope, or to a peak reading AC voltmeter. The output gives 1,1 V for full scale deflection, which is equivalent to 36,6 mV/graduation on the 1521. When both static and dynamic strains are present the 1521 will indicate the static strain, while the AC voltmeter will measure the amplitude of the ripple on the DC level (peak measurement) thereby showing the dynamic strain.

Similar measurements can be made with the Deviation Bridge Type 1519, but its higher excitation frequency can accentuate any capacitive unbalance.

Voltmeter Type 2425

The Electronic Voltmeter Type 2425 can be used on the DC output of the 1521 (or 1519) to measure dynamic strain, but this voltmeter also has several features that make it suitable for measuring dynamic strain alone. Fig.9.3 shows two measuring arrangements that can be used to indicate strain on a 2425.



9.3a. Measurement with Wheatstone bridge

9.3b. Measurement with simplified bridge

Fig.9.3. Measurement of dynamic strain using a Voltmeter Type 2425

Excitation of either of the bridge circuits with a DC voltage (for example a 1,5 V battery) will give no indication on the AC meter when there is no dynamic loading present on the specimen. As soon as a dynamic load is applied, the voltmeter detects the superimposed AC that appears on the DC level, and the peak level of this ripple is proportional to the dynamic strain present.

The 2425 can measure peak levels and it also has a Peak Hold function to capture transients. Other features of the 2425 that help to make it suitable for strain measurement are its frequency range down to 0,5 Hz, and its most sensitive voltage range of 1 mV full scale deflection, which gives 20 μV/scale graduation. One feature that can be a drawback is the very wide frequency range up to 0,5 MHz with no facilities for filtering out high frequency noise. The measuring bridge circuits operate quite well as antennae, so thorough screening and grounding is very important in reducing pick-up and noise. If it should prove very difficult to eliminate high frequency noise from the system, its effects can be greatly reduced by making the measurements in the RMS mode, and then multiplying the measured levels by the Crest Factor (i. e. by 1,4 for a sinusoidal loading).

The dynamic strain level is given by equation 6:8,

$$\varepsilon = \frac{4\,V}{E\,k}$$

where

V = potential measured by the voltmeter
E = the excitation voltage
k = Gauge Factor

Note that the Gauge Factor used here is actually the bridge sensitivity factor.

Consider the example where only one active gauge is used with k = 2, and the excitation voltage is 1,5 V. Putting these values into the equation and taking V = 1 mV (full scale deflection)

$$\varepsilon_{fsd} = \frac{4 \times 1 \times 10^{-3}}{1{,}5 \times 2}$$

$$= 1{,}333 \times 10^{-3}\ \varepsilon = 1333\ \mu\varepsilon\ (fsd)$$

Which gives 26,66 $\mu\varepsilon$/scale graduation.

For a Full-bridge with bridge sensitivity factor 8, and 3,0 V excitation voltage the equation gives 166 $\mu\epsilon$ full scale deflection and 3,33 $\mu\epsilon$/scale graduation.

Voltmeter 2425 and Conditioning Amplifier Type 2626

As previously mentioned, noise pick-up can be a problem when the 2425 is used alone, and if the system noise is higher than the zero load out-of-balance level, accurate calibration will be impossible. An improvement in the signal-to-noise ratio can be obtained when the signal from the measuring bridge is passed through a preamplifier before being fed to the Voltmeter. A low noise preamplifier suitable for this purpose is the Conditioning Amplifier Type 2626, which is normally used as an accelerometer preamplifier. At the price of some minor circuit complication, several advantages are gained from adding a 2626 to the arrangement with a 2425. Up to 60 dB of amplification is available from the Conditioning Amplifier, and progressive high and low pass filtering can be selected in steps to limit the width of the measuring band. Furthermore, the three digit sensitivity adjustment allows the gain to be set to give a direct reading of strain on the Voltmeter scale.

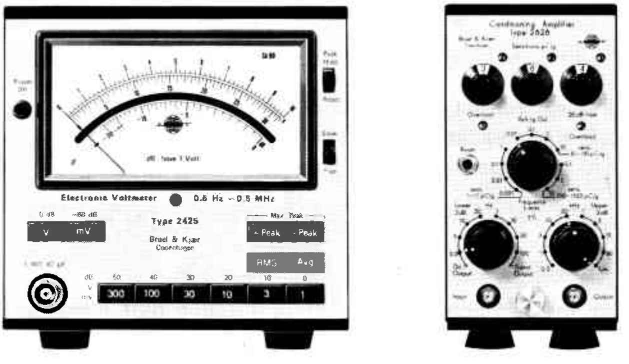


Fig.9.4. Type 2425 and Type 2626

The minor complication is the fact that the 2626 is a charge amplifier, so a 1 nF capacitor has to be put in series with the bridge to give a charge source (in effect this constitutes a dummy accelerometer). Fig.9.5 illustrates the arrangement.

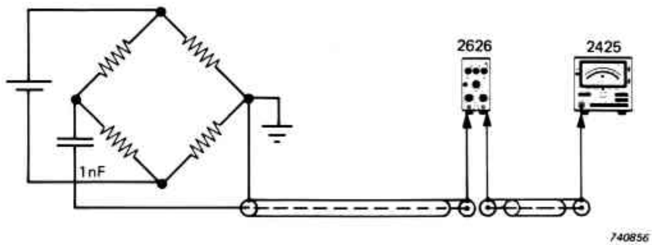


Fig. 9.5. Arrangement for measuring strain using a Conditioning Amplifier Type 2626 and a Voltmeter Type 2425

Setting-up and adjustment is best explained by means of the following examples. Consider a simple case first where the bridge contains two active and two dummy gauges with k = 2 to give a bridge sensitivity for the arrangement of 4, and where the excitation voltage is supplied by a 1,5 V battery. The limiting frequencies are adjusted to cover only the frequency range of interest. Setting the Volt/g Out to position "1" in the range 1 — 11 pC/g, and keeping the Sensitivity set to a simple figure of 1,00 a dynamic level of 220 mV peak is recorded by the Voltmeter, now what does this 220 mV mean in terms of strain?

Because the output was set to 1 V/g, each volt recorded by the 2425 is the equivalent of 1 g measured by an accelerometer. Further, when the source impedance contains 1 nF a Sensitivity of 1,00 pC/g will give the equivalent of 1 g for every millivolt coming from the source (1 mV/g), so that the Voltmeter shows 1 V for every 1 mV coming from the bridge (i. e. 60 dB amplification).

Using equation 6:8 again,

$$\varepsilon = \frac{4\,V}{E\,k}$$

$$= \frac{4 \times 0{,}220 \times 10^{-3}}{1{,}5 \times 4}$$

$$= 0{,}147 \times 10^{-3}\,\varepsilon$$

$$= 147\,\mu\varepsilon$$

Another example showing the use of the Sensitivity adjustment, with all parameters as before, except that Sensitivity is adjusted to 1,50 pC/g (i. e. numerically the voltage), the Voltmeter will indicate 147 mV (numerically the strain). This is because the output is 1 V/g from the 2626, while its input is 1,5 mV/g giving 1 V indication for each 1,5 mV from the bridge.

Hence
$$\varepsilon = \frac{4 \times 0{,}147 \times 10^{-3}}{1{,}5 \times 4} \times 1{,}5$$
$$= 0{,}147 \times 10^{-3}\ \varepsilon$$
$$= 147\ \mu\varepsilon$$

Gauge factors other than 2 can also be compensated for by the Sensitivity adjustment. Taking the previous example again but using gauges with a Gauge Factor 2,2 (i. e. bridge sensitivity factor with two active gauges will be 4,4), if the Sensitivity is adjusted to 1,5 × 2,2/2 = 1,65 pC/g, the output from the 2626 will still be 1 V/g, but the input will be 1,65 mV/g, which gives 1 V indication for each 1,65 mV from the bridge, and the Voltmeter will still indicate 147 mV.

$$\varepsilon = \frac{4 \times 0{,}147 \times 10^{-3}}{1{,}5 \times 4{,}4} \times 1{,}65$$
$$= 0{,}147 \times 10^{-3}\ \varepsilon$$
$$= 147\ \mu\varepsilon$$

Vibration Meter Type 2511

Another instrument that can be used to measure dynamic strain has most of the features of the previous system combined in one instrument, the General Purpose Vibration Meter Type 2511, and adds the possibility of frequency analysis of the strain using an external filter. When the simplified bridge shown in Fig.9.3 is used, a separate battery

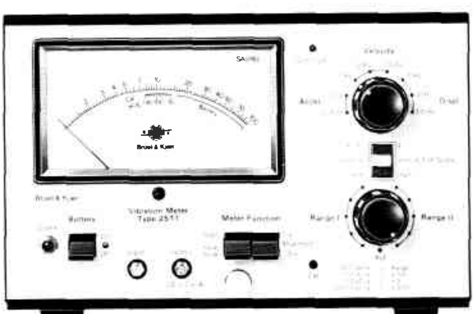


Fig.9.6a. Type 2511

Fig. 9.6b. Type 1621

will not be required as the 28 V DC supply from the 2511 can often give sufficient current to excite the bridge. Fig.9.7 suggests a suitable arrangement that includes provision for frequency analysis with a Tunable Band Pass Filter Type 1621.

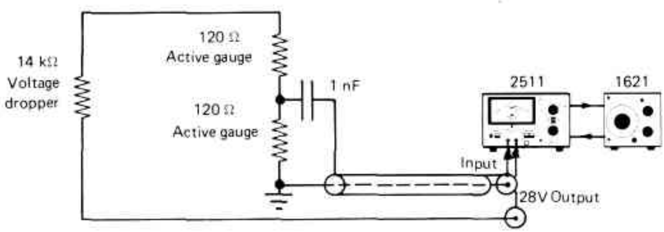


Fig. 9.7. Arrangement for measurement and analysis of strain using a General Purpose Vibration Meter Type 2511 and a Tunable Band Pass Filter Type 1621

When 120 Ω strain gauges are used together with a 14 kΩ voltage dropper, the total resistance in the circuit will be given by,

$$R_T = 14000 + 120 + 120$$

$$= 14{,}24\ k\Omega$$

With a nominal supply voltage of 28 V the current requirement will be,

$$i = \frac{28}{14{,}24}\ mA$$

$$= 1{,}94\ mA$$

This will not overload the 28 V output which has a current limit of 2 mA, so the voltage applied to each gauge will be,

$$E = 1{,}94 \times 120\,mV$$
$$= 0{,}233\ V$$

The 2511 operates as a voltmeter when a 1 nF capacitor is connected in series with the input voltage, so that when the instrument has been calibrated for 100 pC/g accelerometers each "g" indicated in the Acceleration mode represents 100 mV (i.e. 10 g/V). Bearing in mind that the 2511 measures peak-to-peak levels, a Scale factor can be derived from the excitation voltage, the g/V relationship, and the bridge Gauge Factor by using equation 6:8 again.

$$\varepsilon = \frac{4\,V}{E\,k}$$

Where

V = peak measured voltage
E = bridge excitation voltage
k = bridge sensitivity factor

Now $$V = \frac{V_{p-p}}{2}$$

And $$V_{p-p} = 10\,g_{p-p}$$

So that $$V = \frac{10\,g_{p-p}}{2}$$
$$= 5\,g_{p-p}$$

Now rearranging equation 6:8 to get the relationship between measured "g" and strain,

$$\varepsilon = \frac{4 \times 5\,g_{p-p}}{E\,k}$$
$$\frac{g}{\varepsilon} = \frac{E\,k}{20}$$

And putting in values from the arrangement shown in Fig.9.7 with two active gauges giving a bridge sensitivity factor k = 4, and with excitation voltage = 2 × 0,233,

Scale Factor $$\frac{g}{\varepsilon} = \frac{2 \times 0{,}233 \times 4}{20}$$
$$= 0{,}0932\ g/\varepsilon$$

For example if the meter indicates 0,0018 $g_{p\text{-}p}$ with this measuring arrangement,

$$\varepsilon = 0{,}0018 \times 0{,}0932$$
$$= 0{,}000168\ \varepsilon$$
$$= 168\ \mu\varepsilon$$

When a Tunable Filter 1621 is connected as the external filter for the Vibration Meter, the signal from the measuring bridge can be frequency analyzed. Further, the narrow band filtering available gives a big improvement to the signal-to-noise ratio of the set-up.

Measuring Amplifiers and Analyzers

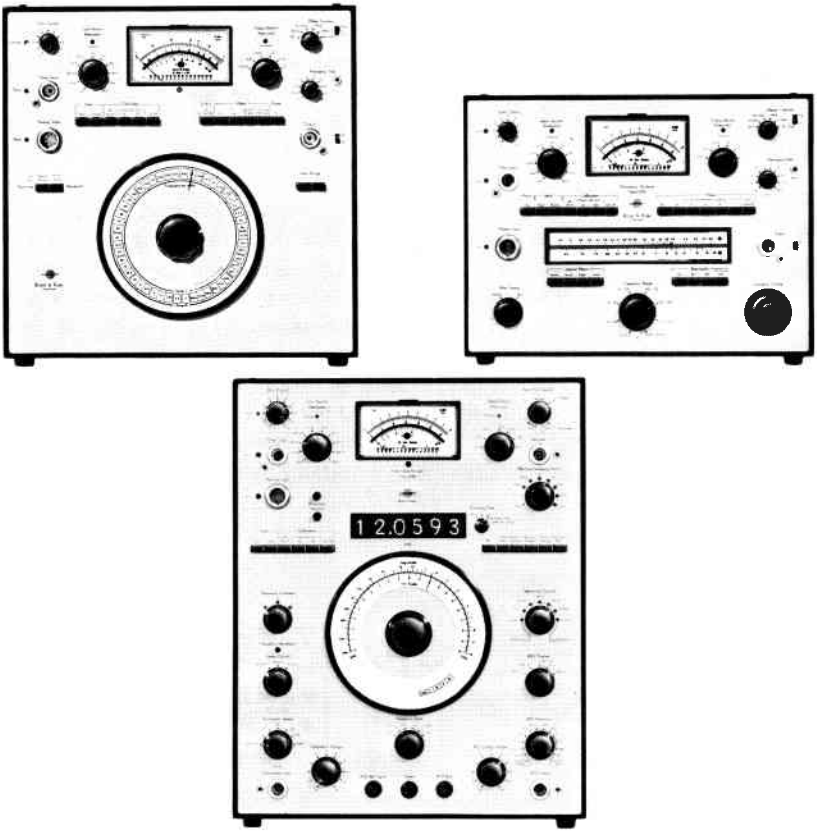


Fig.9.8. Type 2114, Type 2120 and Type 2010

One final group of instruments that can be used for measurement of dynamic strain are the B & K Measuring Amplifiers and Frequency Analyzers. The Measuring Amplifiers Type 2606, Type 2607, Type 2608, and Type 2609 can be used in exactly the same way as the Voltmeter, but as the 2608 and 2609 only record RMS levels the Crest Factor of the applied loading must be known when these two instruments are to be used. The Type 2609 has a most sensitive voltage range of 100μV for full scale deflection, and can measure in the audio frequency range 20 Hz to 20 kHz. The other Measuring Amplifiers can measure 10μV with full scale deflection and have a frequency range from 2 Hz to 200 kHz.

When one of the Measuring Amplifiers (not 2609) is used with an external filter, or when one of the Frequency Analyzers Type 2010, Type 2113, Type 2114, Type 2120, or Type 2121 is used, the signal from the measuring bridge can be filtered to improve the signal-to-noise ratio, or to make a frequency analysis of the dynamic strain levels. Again there will be some restrictions as the 2010 and 2121 can only measure RMS levels.

All analyzers can make linear measurements in the frequency range 2 Hz to 200 kHz, and have a maximum sensitivity that gives 10μV full scale deflection. The 2113 and 2121 can make frequency analysis of the strain signals in the range 20 Hz to 20 kHz, the 2113 with octave or third-octave, and the 2121 with third-octave, 10%, 3%, or 1% constant proportional bandwidths. The 2120 can make narrow band (third-octave, 10%, 3%, or 1%) analysis in the range 2 Hz to 20 kHz, while the 2114 covers the frequency range 2 Hz to 180 kHz in octaves or third-octaves. The 2010 can make frequency analysis over the range 2 Hz to 200 kHz, with constant bandwidths selectable between 1000 Hz and 3,16 Hz.

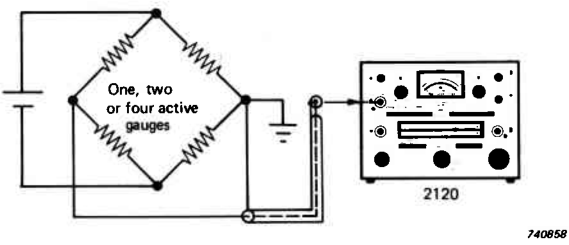


Fig.9.9. Dynamic strain measurement arrangement using a Frequency Analyzer as the measuring instrument

The method of zero calibration, and measurement will be the same as when the Voltmeter is used. For example, when one active gauge is used with Gauge Factor 2, and the excitation voltage is 1,5 V, the strain for full scale deflection in the most sensitive range (10 μV fsd),

$$\varepsilon_{fsd} = \frac{4\,V}{E\,k}$$

$$= \frac{4 \times 10 \times 10^{-6}}{1{,}5 \times 2}$$

$$= 13{,}3\ \mu\varepsilon\ (fsd)$$

Which gives 0,266 $\mu\epsilon$/scale graduation (0 to 10 V scale).

CHAPTER 10
SEMICONDUCTOR STRAIN GAUGES

Semiconductor strain gauges are made from a thin strip conductor that has been mechanically, or photo-chemically cut from a single crystal of silicon or germanium. These materials have a high piezo-resistive effect so that the electrical conductivity is very dependent upon the applied strain, therefore high gauge factors can be obtained that are 50 to 100 times greater than with metal gauges. The most common applications for semiconductor gauges are those that take advantage of the high gauge factors available. Very low strain levels can be measured, or high bridge outputs can be obtained to drive relays for example.

The crystal material is prepared by "doping" silicon or germanium with an accurately predetermined amount of an impurity to give the required gauge characteristics. Semiconductor gauges can be obtained as P-types with positive gauge factors, or as N-types with negative gauge factors,

Fig.10.1 and Fig.10.2 show typical resistance change/strain curves for a P-type and an N-type gauge respectively. It will be observed that the slope of the curves (i. e. the gauge factor k) is not a constant for semiconductor gauges, but that it varies with the applied strain level.

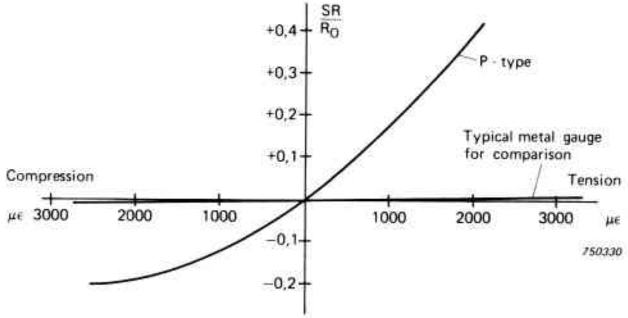


Fig.10.1. Typical strain sensitivity curve for a P-type semiconductor gauge

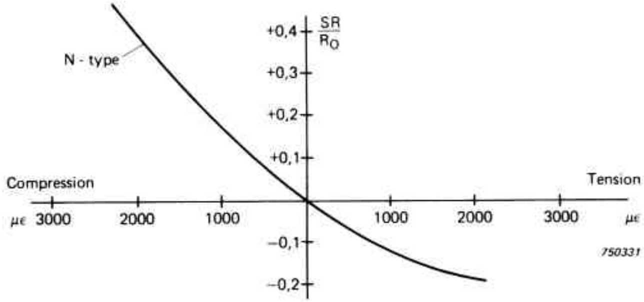


Fig.10.2. Typical strain sensitivity curve for an N-type semiconductor gauge

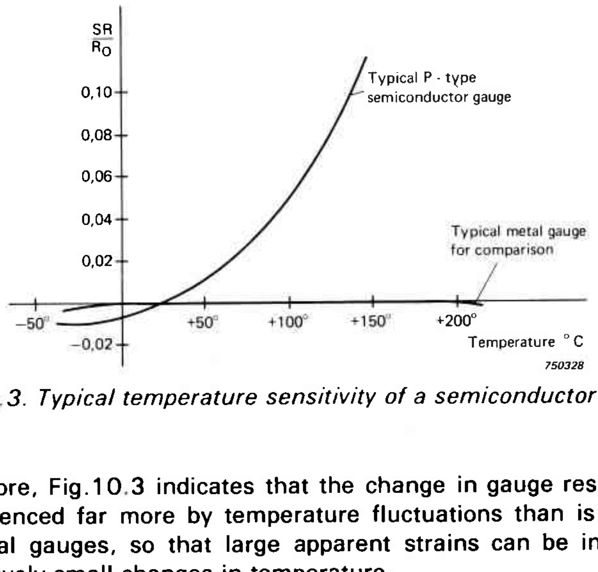


Fig.10.3. Typical temperature sensitivity of a semiconductor gauge

Furthermore, Fig.10.3 indicates that the change in gauge resistance is also influenced far more by temperature fluctuations than is the case with metal gauges, so that large apparent strains can be induced by comparatively small changes in temperature.

A further complication is caused when the strain gauge crystal is cemented onto the backing, the resistance changes due to shrinkage induced strains as the adhesive drys, and when the gauge is mounted on the test specimen, the resistance changes yet again. (It might be noted that this latter effect also occurs with metal gauges, but even though the mounted gauge may be in a prestressed condition, the gauge factor for the metal gauge remains constant. Further, as the gauge factor for the metal gauge is only one fiftieth of that for the semiconductor gauge, the change in resistance of the metal gauge due to shrinkage effects will be fifty times smaller).

At the first sight of all these factors that influence the resistance of the gauge — and hence its gauge factor — it might appear doubtful whether accurate measurements could ever be obtained, however the situation is not quite as bad as it looks. As shown in Fig.10.4 and Fig.10.5, for a single active semiconductor gauge employed where tem-

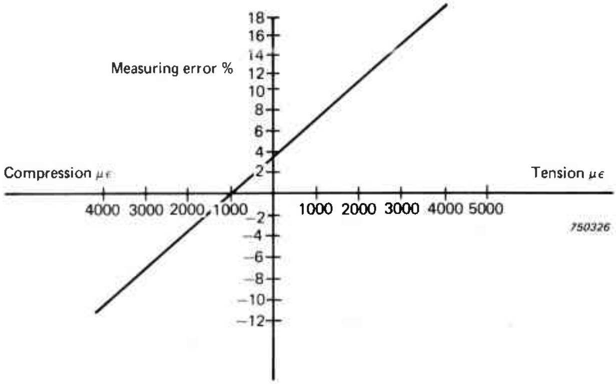


Fig.10.4. Typical accuracy of measurement when the nominal gauge factor k_o is used with a single active P-type gauge

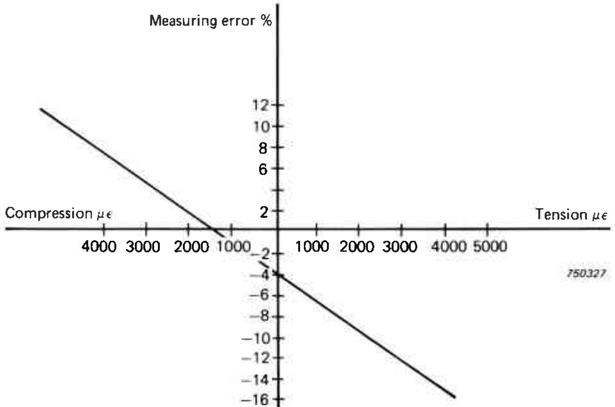


Fig.10.5. Typical accuracy of measurement when the nominal gauge factor k_o is used with a single active N-type gauge

perature variations are less than ± 10°C, use of the nominal gauge factor gives a typical measuring accuracy better than ± 10% at strain levels up to + 2000 $\mu\epsilon$, and down to —4000 $\mu\epsilon$, ± 5% between + 500 $\mu\epsilon$ and —2500 $\mu\epsilon$.

For most measuring purposes where very small levels are to be measured, and where temperatures can be kept stable, this order of accuracy will be sufficient. Further calibration will be unnecessary, and the information in the rest of this chapter can be ignored.

However, where large temperature variations are encountered and good accuracy required, several correction techniques can be applied. The best compensation is obtained where a matched pair of one P-type and one N-type (or a double resistor P-N type) is used in a half bridge arrangement, or two of each are connected in full bridge. Complete temperature compensation is obtained, and the nominal gauge factor is accurate and constant for the whole operating range of the gauges.

Compensation gauges can be used in half or full bridge connections, in the same way as for metal gauges, to eliminate temperature effects completely. When the bridge is composed of all P-type gauges, there will still be a constant overall measuring error of about + 3,5%. With an all N-type bridge the error will be constant at approximately —3,5%. These errors, due to shrinkage as the mounting cement drys, can be corrected by a suitable adjustment of the gauge factor.

Single self-compensating N-type gauges can be obtained that have special temperature characteristics designed to compensate for the temperature expansion of the test material. Typically this type of gauge will have a constant temperature deviation (θ_t) of + 5 or + 6 $\mu\epsilon$/°C when mounted on the specified material. P-type gauges can not be temperature compensated to the same degree, typically "θ_t" will be of the order of + 9 to + 10 $\mu\epsilon$/°C.

A further method of temperature compensation for single active semiconductor gauges uses a constant current excitation, instead of constant voltage. Referring to Fig.10.3 it will be seen that the change in resistance ($\delta R/R_o$) varies from —0,01 to + 0,10 over the temperature range from about —50° to + 150°C, which represents an apparent strain variation of —100 $\mu\epsilon$ to + 1000 $\mu\epsilon$. Over this range of apparent strain the relationship $\delta R/\epsilon$ can be assumed to be a constant, therefore from the original gauge factor equation $k = \delta R/R\epsilon$, k must be inversely proportional to R. That is, when the resistance R rises with temperature, the sensitivity (gauge factor k) falls. A constant current applied to the gauge causes the excitiation voltage to vary in direct proportion to the gauge resistance. Thus when the temperature increases, the rise in resistance causes a rise in the excitation voltage so that a proportionally greater voltage appears in the measuring link and compensates for the reduced sensitivity of the gauge. In this way the effective gauge factor (= sensitivity) of the gauge can be kept virtually constant through comparatively wide temperature variations.

Accurate determination of strain with semiconductor gauges

If greater measuring accuracy is required from semiconductor gauges, and P-N combinations cannot be used for some reason, a correction must be calculated. In addition to all the sources of error common to strain gauges in general that have been described elsewhere in this book, the nonlinear gauge factor of semiconductor gauges contributes a major part of the measuring error. As described in the previous section, the great sensitivity of semiconductor gauges to temperature can also give considerable measuring errors. However, when the nature of these errors is understood, suitable calibration techniques can be applied so that accurate measurement is possible.

The first point to bear in mind is that the gauge factor quoted by the maker should be considered purely as a nominal value for guidance, valid only for the temperature at which the gauge was calibrated. This is obviously true where the characteristic curve of resistance change against applied strain is of parabolic form, so that the effective gauge factor (the slope) is dependent upon the applied strain level.

When a semiconductor strain gauge is received from the supplier, the following information is usually given for each individual gauge, or pack of gauges:

$R_0 \Omega \pm \%$, resistance of the unstrained gauge at some specified temperature. R_0 can only be measured on an unmounted gauge conductor.
k ±%, nominal gauge factor, i. e., the gauge factor when the conductor resistance is R_0, which is the condition for an unstrained conductor at the calibration temperature. (Herein lies the first major source of error that must be compensated for.)
$\delta k/k\%$ per degree temperature, variation of gauge factor with temperature.

$\theta_t \mu\epsilon$ per degree temperature, variation of apparent strain with temperature.

From this information and a sensitivity curve like those shown in Fig.10.1 and Fig.10.2, the actual strain level can be found. The sensitivity curve should be available from the gauge supplier or manufacturer.

Another typical gauge sensitivity curve is shown in Fig.10.6 which also indicates where measuring errors can occur because of zero shift caused by the cement shrinking as it drys during mounting. The sensitivity curve obtained from the gauge manufacturer will only show the actual curve and the axes that pass through O, (marked $\delta R/R_0$). If the resistance of an installed gauge is measured and compared with the quoted R_0 value, it will be found that with a P-type gauge the resistance of the mounted gauge R_m is about 10 to 12% lower than R_0, and for an N-type gauge the mounted resistance will be from 10 to 12% higher. These changes have occured as the cement dried, and indicate that shrinkage has induced a compressive strain of the order of $1000\mu\epsilon$, (this is quite a typical value for shrinkage strain), so that even though no strain is being applied to the test specimen, the "zero" for the mounted gauge has moved along the curve to point M.

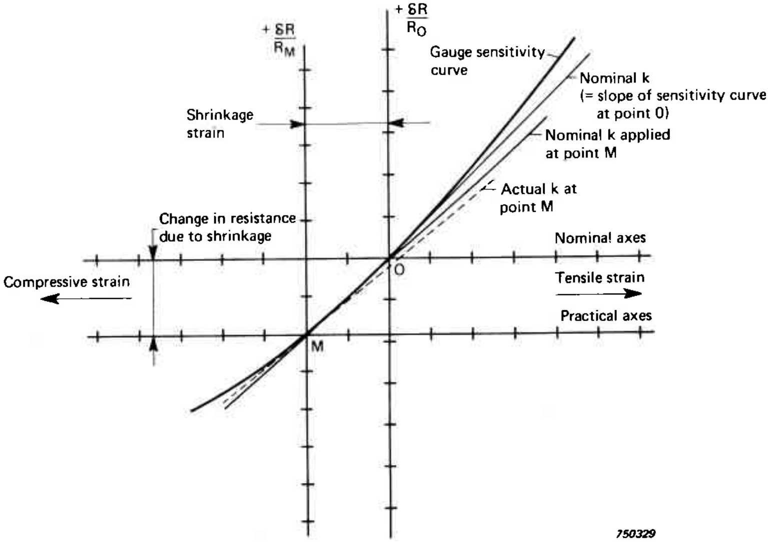


Fig.10.6. Errors due to zero shift during mounting a P-type gauge

The nominal gauge factor k for the gauge is given by the slope of the sensitivity curve where the gauge resistance is equal to R_o and where the gauge conductor is completely unstrained, i. e. the slope at point 0. If the nominal gauge factor is now used when the test piece is strained the $\delta R/R$ term registered by the measuring instrument will refer to the changes in the mounted resistance R_m. This is shown in the figure by the nominal k line passing through the new zero strain point M, so that measuring errors will occur due to the difference between the sensitivity curve and the nominal k line passing through M. These measuring errors are made up from two components, one is the difference between a straight "nominal sensitivity" line and the generally parabolic true sensitivity "curve". This is the factor that causes the slope in the error curves in Fig.10.4 and Fig.10.5. The other component is the difference between the slopes of the "true sensitivity" curve and the nominal sensitivity curve at point "M". This is the difference that causes the measuring error at zero applied strain for single active gauges, or the constant measuring error found when similar types of gauge are used for temperature compensation in half or full bridge connections.

The only really sure method of avoiding errors due to a parabolic sensitivity curve, and errors due to zero shift, is to refer all measured values back to the true gauge sensitivity curve, and to the completely unstrained condition of the gauge conductor. In this way the approximations implied in the assumption of the nominal gauge factor value are eliminated.

Accurate measurement with the Strain Indicator 1526

The following example is suggested as a practical method for making accurate measurements with a single active semiconductor gauge using the Strain Indicator 1526. Similar methods can be used to get accurate strain levels when multi-gauge bridges are used with similar type gauges. The method is divided into four stages as follows: 1, Determination of the resistance of the mounted but unstrained gauge to find the apparent strain due to shrinkage, 2, Measurement of the indicated strain due to the applied mechanical loading of the specimen so that 3, the actual applied strain level can be determined, and 4, Determination of the amount of strain induced in the gauge when measurements are made at a different temperature from the calibration temperature.

The manufacturers data for the gauge used in the example is as follows:

Gauge: P-type silicon,
Nominal gauge factor. $k = 112 \pm 5\%$
Gauge resistance: $R_o = 126\,\Omega \pm 0{,}5\%$
Calibration temperature: $t_o = 20°C$
Temperature sensitivity: $\delta k/k = 0{,}22\%$ per °C
$\theta_t = +10{,}1\,\mu\epsilon$ per °C
Sensitivity curve at 20°C as shown in Fig.10.8, and temperature sensitivity curve as shown in Fig.10.3.

1. Determination of the resistance of the mounted gauge

As all the subsequent strain levels measured will be based upon the value obtained for the resistance of the mounted gauge and will have the same order of accuracy, the resistance must be determined with as much accuracy as possible, but there is no point in greater accuracy than the value given for R_o. When a good quality ohmmeter is available that uses a low voltage excitation, the resistance can be measured directly. The common, inexpensive multi-meter is not suitable for this purpose as its measuring accuracy is typically $\pm$ 5%.

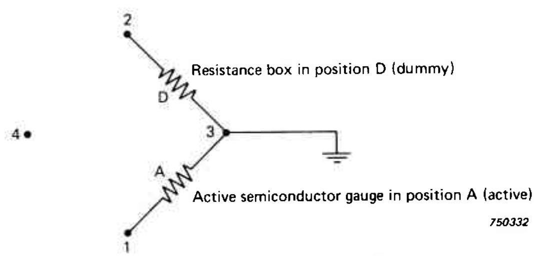


Fig.10.7. Gauge and resistance box connection to the Type 1526 for measurement of gauge resistance

The resistance of the gauge on the unloaded specimen can be determined by using the 1526 as an ohmmeter when an accurate decade resistance box is available. A decade box with an accuracy of 0,5% or better is quite suitable. The active semiconductor strain gauge, and the resistance box should be connected to the 1526 as shown in Fig.10.7.

1. Set the 1526 to "0,3 V" excitation voltage, and "2000 $\mu\epsilon$" Range.

2. Set the resistance box to the nominal gauge resistance value R_o.

3. Set the 1526 gauge factor to "about" 10.

4. Switch the bridge mode to "Half-bridge, Off" to eliminate the balance potentiometer from the bridge circuit.

5. Select "Operate". The display will blink when the resistance of the mounted gauge (R_m) is more than 2% different from R_o. (The 1526 is only designed to balance out 2% on its own potentiometer).

6. Adjust the resistance in the decade box in the direction indicated by the display (— means decrease, + means increase) until first the display stops blinking, and then in 1 Ω steps until the display sign changes. In the example this occured between 110 Ω and 109 Ω steps.

7. Now switching back to the previous ohmic value and then on again so that the display alternates between a small negative and a small positive value, adjust the "Gauge Factor" potentiometer on the 1526 until there is a total of 1000 $\mu\epsilon$ difference between the two values displayed. (The exact value of the gauge factor is not important.) The 1526 has now been adjusted so that it acts like an ohmmeter with a resolution of 1000 digits per ohm at the gauge resistance, and a measuring accuracy comparable to the accuracy of the resistance values in the decade box.

8. The actual resistance of the mounted gauge is given by the settings on the decade box plus the displayed value. In the example these values for R_m were:

$$109\ \Omega + 410\ \mu\varepsilon = 109{,}41\ \Omega$$

and

$$110\ \Omega - 590\ \mu\varepsilon = 109{,}41\ \Omega$$

± % accuracy of decade box, when the ambient temperature at the gauge was 20°C.

Strictly speaking, the out-of-balance of the 1 kΩ resistors forming the other side of the half bridge should also be checked, but as they are garanteed to have less than 50 ppm difference between them, the effect on the display will be very small. Nevertheless, unbalance is checked as follows:

9. Remove the semiconductor gauge and decade box connections from the terminals of the 1526, and replace them with the Calibration Bridge ZR 0013.

10. Without altering any of the other settings, check the displayed value. The position of the switch on the Calibration Bridge is unimportant as it is completely balanced in either setting.

11. Add or subtract the displayed value (according to sign) from the gauge resistance. It should be no more than a few thousandths of an ohm. In the example the unbalance was $+ \underline{1}\,\mu\epsilon$, equivalent to 0,001 Ω which should be subtracted from the resistance value. In this particular example it has no significant effect on the resistance value found.

The difference in resistance between R_m and R_o

$$\delta R = 109{,}41 - 126 = -16{,}59\ \Omega$$

so that
$$\frac{\delta R}{R_o} = -\frac{16{,}59}{126} = -0{,}1315$$

From the gauge sensitivity curve (Fig.10.8) the shrinkage strain is found to be $-1210\,\mu\epsilon = -0{,}001\,21$ strain

The slope of the sensitivity curve at M in terms of R_m is given by

$$k_m = \frac{\delta R}{R_m \varepsilon} \tag{10:1}$$

where
$$\frac{\delta R}{R_m} = \frac{\delta R}{R_o} \times \frac{R_o}{R_m} \tag{10:2}$$

so that
$$k_m = \frac{2 \times 0{,}1315 \times 126}{126 \times 109{,}41 \times 2500}$$

$$k_m = 96$$

This is the average k_m over a short portion of the curve having M at its centre.

A similar method of resistance measurement may be employed using an accurately calibrated potentiometer.

2. Measurement of the indicated strain

The knowledge of the amount of shrinkage strain there is on the gauge, and of the actual gauge factor existing at the point where the test piece is unstrained open up several new possibilities.

First, if the value k_m just calculated for the gauge factor is set on the 1526 (actually $k_m/10$ is set), or if the gauge factor on the 1526 is set to 10 and the k_m value used in the formula

$$\frac{\textit{Displayed value} \times 10}{k_m} = \textit{microstrain} \qquad (10:3)$$

the results obtained will have an accuracy (typically) better than ± 5% over the measuring range between ± 1500 $\mu\epsilon$, plus the accuracy to which the resistance of the unstrained mounted gauge was determined. This is because the error curves in Fig.10.4 and Fig.10.5 have been moved to pass through zero while retaining their slopes.

For most practical measuring systems, the accuracy obtained with this method will be sufficient. However, if a higher degree of accuracy is still required, one of the alternative methods described in section 3 should be followed.

3. Determination of the actual strain level

The method that has just been used to determine the mounted resistance of the gauge can be used again to find the resistance of the strain gauge when the test piece is subjected to an applied static mechanical strain. A new $\delta R/R_0$ value can be found that yields a new strain value from the sensitivity curve which can be added to the shrinkage strain to give the actual change in strain level caused by the applied mechanical strain.

Returning to the worked example, when the strain was applied to the test specimen, the new resistance found by this method was 149,22 Ω.

Hence $\qquad \delta R = 149{,}22 - 126 = +23{,}22\ \Omega$

and $\qquad \dfrac{\delta R}{R_o} = \dfrac{+23{,}22}{126} = +0{,}184$

Which, according to the sensitivity curve, is equivalent to a strain level of + 1570 $\mu\epsilon$. Therefore the change in strain level between the unstrained state of the test piece and the strained state,

= 1570 tens — (—1210 comp)

which gives a total of $+2780\mu\epsilon$ change in level from the unstrained state, i.e., the mechanical strain level applied to the test piece is $2780\mu\epsilon$ tensile, with an accuracy equivalent to the resistance determination.

The foregoing method for calculating the actual applied mechanical strain can be used for measurements up to the maximum permissible level for the gauge, (4000 to $5000\mu\epsilon$). However, as the method is rather tedious, especially where many strain measurements have to be determined, and as it is not suitable for measuring dynamic strain either, the following method is generally recommended for measurement of static or dynamic levels up to about $2000\mu\epsilon$ (with k = 100).

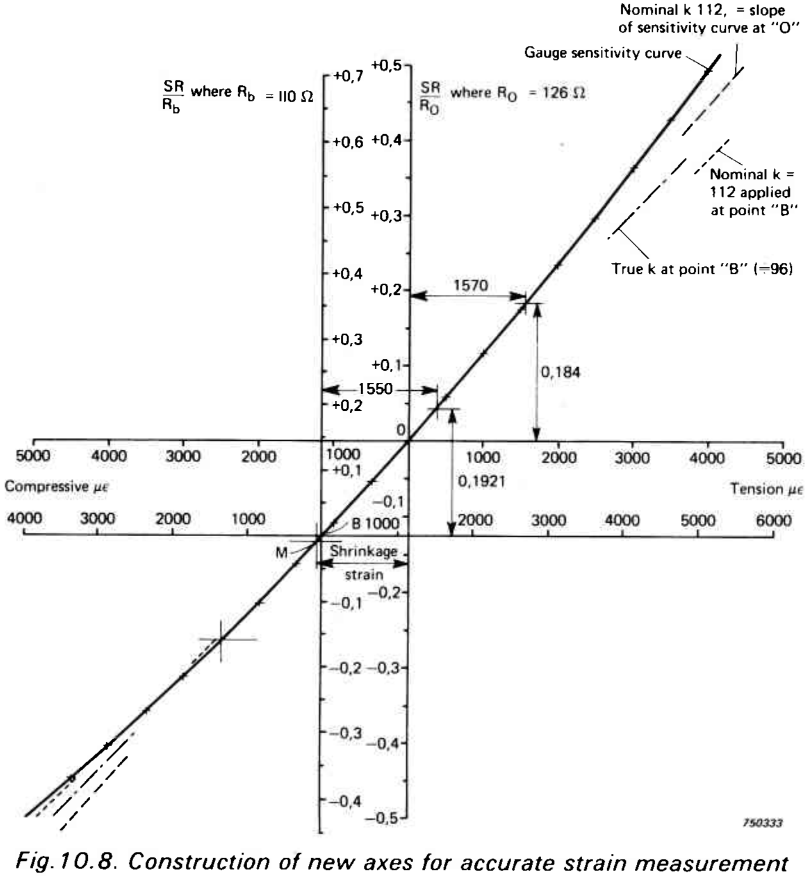


Fig. 10.8. Construction of new axes for accurate strain measurement

When the mounted resistance of the semiconductor gauge has been determined, the decade resistance box or calibrated potentiometer can be removed and replaced in the half bridge by a resistor.

Ideally the resistance of the dummy should be exactly the same as the value found for R_m, so that it will not be necessary to use the built-in balance potentiometer in the 1526. When it is not possible to obtain or fabricate the exact resistance value required, the nearest approximation (less than 2% away) can be used. The resistance of the dummy gauge (R_d) must also be known with a high degree of accuracy because now when the half bridge is balanced, the resistance of the active and the dummy arms will have the same value (R_b) which will be the datum resistance for all subsequent measurements.
R_b is given by,

$$R_b = \frac{R_m + R_d}{2} \ \Omega \qquad (10{:}4)$$

A new pair of axes can be drawn through point B where $\delta R/R_o$ crosses the sensitivity curve at a δR value equal to $R_o - R_b$. (This will be point M when the dummy and active gauges have identical resistance = R_m). The scale of the new strain axis through B has exactly the same increments and size as the strain axis through 0. It has merely been moved an amount equivalent to the strain level indicated by a resistance change of $R_o - R_b$. The $\delta R/R_b$ axis will be scaled with resistance change increments referred to R_b, and these are given by a relationship similar to that indicated by equation 10:2,

Thus
$$\frac{\delta R}{R_b} = \frac{\delta R}{R_o} \times \frac{R_o}{R_b} \qquad (10{:}5)$$

The resistor used as the dummy gauge in the worked example was found to have a resistance of 110,59 Ω ± 0,5%.

therefore
$$R_b = \frac{109{,}41 + 110{,}59}{2}$$
$$= 110{,}00\ \Omega$$

Hence
$$\frac{\delta R}{R_o} = \frac{110 - 126}{126}$$
$$= -0{,}127$$

Which is where the new strain axis crosses the sensitivity curve at point B. One unit of the $\delta R/R_b$ scale will be given by,

$$\frac{\delta R}{R_b} = \frac{\delta R}{R_o} \times \frac{126}{110}$$
$$= 1{,}145 \frac{\delta R}{R_o}$$

i. e., the increments on the $\delta R/R_o$ scale are 1,145 times greater than the increments on the $\delta R/R_b$ scale.

Axes with these scales have been drawn through point B on the sensitivity curve shown in Fig.10.8.

The gauge factor on the 1526 should now be reset to 10,00 and the half bridge balanced using the built-in balance potentiometer. The specimen is loaded with a static or dynamic strain, and an indication obtained on the digital display, or output in the normal way. The quantity displayed will be $\delta R/R_b$ in units of $\mu\Omega/\Omega \times 10$ because the gauge factor set is 10. The displayed value can readily be converted to a resistance value just by moving the decimal place. Then the actual strain applied to the test specimen can be read from the sensitivity curve, remembering to read from the axes passing through B.

Returning once again to the example, when the strain was applied, the display indicated $+19210\mu\epsilon$, which means that the strain gauge had undergone a resistance change of $+192100\mu\Omega/\Omega$ (see above), so that $\delta R/R_b$ equals $+0{,}1921$.

Applying this resistance change to the sensitivity curve, it is seen that it will be brought about by an applied strain of $+1550\mu\epsilon$. This measurement is accurate within $\pm 1{,}0\%$, being the sum of the accuracies of the determination of the gauge resistance and of the dummy resistance.

Up until now, all the measurements and corrections have assumed that the temperature at the measuring point remains constant. Methods for calculation the effect of temperature variations are described in section 4.

4. Determination of apparent strain due to temperature variation

The change in resistance of a semiconductor strain gauge that is detected by the measuring instrument is made up from a component caused by the applied mechanical strain, and from a component due to temperature induced apparent strain.

Total strain = Mechanical strain + Thermal strain

When the strain gauge in the example had its temperature raised to 100°C while it was subjected to a mechanically applied strain, the display indicated 17 600 $\mu\epsilon$. This value represents a change in resistance of 176 000 $\mu\Omega/\Omega$, so that

$$\frac{\delta R}{R_b} = 0{,}176$$

and $R_b = 110\,\Omega$

therefore
$$\delta R = 110 \times 0{,}176 = 19{,}35\,\Omega \qquad (10{:}6)$$

and the resistance of the loaded strain gauge at 100°C,

$$R_{L(100)} = 110 + 19{,}35 = 129{,}35\,\Omega$$

From the temperature curve Fig. 10.3,

$$R_o \text{ at } 100°C = R_o \text{ at } 20°C \times 1{,}052$$
$$= 126 \times 1{,}052$$
$$R_{o(100)} = 132{,}6\,\Omega$$

And
$$\frac{R}{R_{o(100)}} = \frac{129{,}35 - 132{,}6}{132{,}6}$$

$$= -0{,}0245$$

From the original gauge sensitivity curve (with axes through 0), the $\delta R/R_0$ value —0,0245 would have been produced by the application of —200 $\mu\epsilon$.

The total change in strain is the sum of the shrinkage strain plus the loaded strain,

$$= 1210 + (-200)$$
$$= 1010\,\mu\epsilon$$

This value is made up from the apparent strain due to the difference in expansion between the test object and the strain gauge (thermic strain) and the mechanical component caused by the applied load. Thermic strain can be determined from,

$$
\begin{aligned}
\epsilon_t &= \theta_t \times \textit{temperature difference} \\
&= 10{,}1 \times (100 - 20) \\
&= 808\,\mu\epsilon
\end{aligned}
$$

Therefore the strain due to mechanical loading,

$$
\begin{aligned}
\epsilon_m &= 1010 - 808 \\
&= 202\,\mu\epsilon
\end{aligned}
$$

Again, the accuracy of this calculated value will depend upon the accuracy with which the resistances were determined.

High strain levels can be measured, if necessary, when a resistance box is used as the dummy. If the bridge output goes outside the range of the 1526 (flashing display), the value of the dummy resistance can be altered until the display again shows a constant value. The change in the value of the dummy will give a new value to the resistance at the balance condition. The new balance value can be found from,

$$
R_{balance} = \frac{R_{gauge} + R_{dummy}}{2}
$$

and it should be inserted in Equation 10:6 in place of the 110 Ω, while the value from the display gives a new $\delta R/R_b$.

Bibliography

The following books will be useful for engineers who wish to make further study in strain gauge techniques.

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Wilhelm Ernst und Sohn, Berlin.

Concise Guide to Structural Adhesives, Guttman,
Reinhold, New York.

Adhesive Materials, Their Properties and Usage, Katz,
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Wilhelm Pansegrav Verlag, Berlin.

APPENDIX I

Derivation of expressions for the position of the principal axes referred to a set of arbitrarily positioned axes

Consider Fig.A1.1, it shows a pair of axes O—X and O—Y with a gauge length O—A = l making an angle α with the X axis. If a strain is applied in the X direction so that the point A moves to A', for small changes in α, the strain in the direction α will be:

$$\varepsilon_\alpha = \frac{\delta l}{l}$$

where $$l = \frac{x}{\cos\alpha}$$

and $$\delta l = \delta x \cos\alpha$$

so that $$\varepsilon_\alpha = \frac{\delta x \cos\alpha \cos\alpha}{x}$$

$$= \frac{\delta x \cos^2\alpha}{x} \qquad \text{(A1:1)}$$

hence $$\varepsilon_\alpha = \varepsilon_x \cos^2\alpha \qquad \text{(A1:2)}$$

Similarly, if a strain is applied in the y direction the expression

$\varepsilon_\alpha = \varepsilon_y \sin^2\alpha$ can be derived (A1:3)

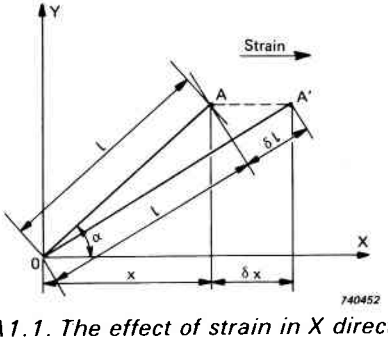

Fig.A1.1. The effect of strain in X direction

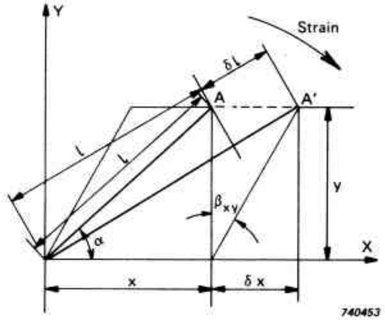

Fig.A1.2. The effect of shear strain

Finally, Fig.A1.2 shows the effect of applying a shearing strain β_{xy} to the gauge length l. The strain in the direction α will be:

$$\varepsilon_a = \frac{\delta l}{l} = \frac{\delta x \cos^2 a}{x}$$

similar to equation A1:1 above,

now $\delta_x = y \tan\beta_{xy} \doteqdot y\beta_{xy}$ (for small angles β_{xy})

therefore $$\varepsilon_\alpha = \frac{y\beta_{xy}\cos^2 a}{x}$$

but $$\frac{y}{x} = \tan a,$$

and $\tan a \cos a = \sin a$

therefore $\varepsilon_a = \beta_{xy} \sin a \cos a$ (A1:4)

If all these strains are applied simultaneously, the effect on the gauge length l can be found by adding the components algebraically, thus

$$\varepsilon_a = \varepsilon_x \cos^2 a + \varepsilon_y \sin^2 a + \beta_{xy} \sin a \cos a$$ this is equation 2:5 (A1:5)

ϵ_α can be measured with a strain gauge, so that if measurements are made at three different angles, to give three different values for ϵ_α and α, the three unknowns can be calculated by solving three equations simultaneously, so that the strains at the measuring point are defined in terms of the arbitrary axes X and Y.

Further, if the above equation (A1:5) is now rewritten in terms of the double angle 2α:

$$\varepsilon_\alpha = \varepsilon_x \left(\frac{1+\cos 2\alpha}{2}\right) + \varepsilon_y \left(\frac{1-\cos 2\alpha}{2}\right) + \frac{\beta_{xy}\sin 2\alpha}{2}$$

$$\varepsilon_\alpha = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\alpha + \frac{\beta_{xy}\sin 2\alpha}{2} \qquad \text{(A1:6)}$$

and then differentiated with respect to α, the angle of the principal planes can be found:

$$\frac{d\varepsilon_x}{d\alpha} = \frac{-2(\varepsilon_x - \varepsilon_y)}{2}\sin 2\alpha_p + \frac{2\beta_{xy}\cos 2\alpha_p}{2} = 0$$

$$\frac{\sin 2\alpha_p}{\cos 2\alpha_p} = \tan 2\alpha_p = \frac{\beta_{xy}}{\varepsilon_x - \varepsilon_y} \qquad \text{this is equation 2:6 (A1:7)}$$

hence the principal strains (ϵ_{max} , ϵ_{min} , and β) acting on the principal planes, can be calculated by substitution back into equation A1:5, and the stresses found by a further substitution into equation 2:4.

Appendix 2 contains worked examples using these equations, and also gives a graphical method devised by Mohr that can be used to find the magnitudes and directions of the principal strains at a point.

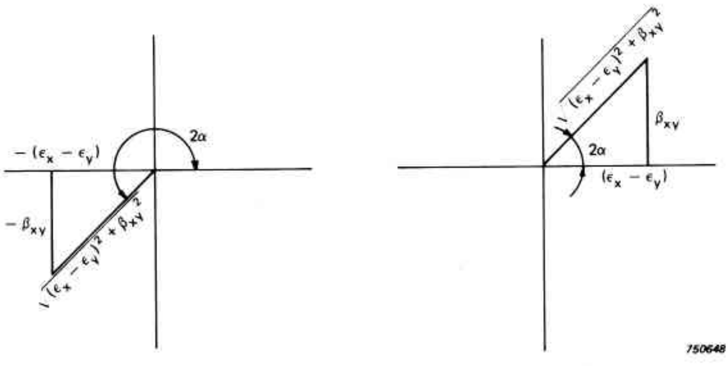

Fig.A1.3. Triangles that contain the angle 2α

The trigonometrical relationships used in equation A1:6 can be found from equation A1:7. A triangle can be drawn that contains the angle 2α, as well as sides with lengths of β_{xy} and $\epsilon_x - \epsilon_y$. Two such triangles are shown in Fig.A1.3. It will be seen that the hypotenuse must be,

$$= \sqrt{(\varepsilon_x - \varepsilon_y)^2 + \beta_{xy}^2}$$

$$\text{so that } \sin 2\alpha = \pm \frac{\beta_{xy}}{\sqrt{(\varepsilon_x-\varepsilon_y)^2+\beta_{xy}^2}}$$

$$\text{and } \cos 2\alpha = \pm \frac{(\varepsilon_x-\varepsilon_y)}{\sqrt{(\varepsilon_x-\varepsilon_y)^2+\beta_{xy}^2}}$$

These values can be substituted into equation A1:6 to yield expressions for the maximum and minimum principal strains. First using the positive values to obtain the maximum principal strain as follows,

$$\varepsilon_{max} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\,\frac{(\varepsilon_x-\varepsilon_y)}{\sqrt{(\varepsilon_x-\varepsilon_y)^2+\beta^2_{xy}}} + \frac{\beta_{xy}}{2}\,\frac{\beta_{xy}}{\sqrt{(\varepsilon_x-\varepsilon_y)^2+\beta^2_{xy}}}$$

$$= \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{(\varepsilon_x-\varepsilon_y)^2+\beta^2_{xy}}{2\sqrt{(\varepsilon_x-\varepsilon_y)^2+\beta^2_{xy}}}$$

$$\varepsilon_{max} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\sqrt{(\varepsilon_x-\varepsilon_y)^2+\beta^2_{xy}}}{2} \qquad \text{(A1:8)}$$

$$\text{Similarly } \varepsilon_{min} = \frac{\varepsilon_x+\varepsilon_y}{2} - \frac{\sqrt{(\varepsilon_x-\varepsilon_y)^2+\beta^2_{xy}}}{2} \qquad \text{(A1:9)}$$

Now standard equations can be developed for the more commonly used strain rosettes, to give the magnitude and direction of the principal strains in terms of the strains measured by the individual gauge grids.

The Delta Rosette

Delta rosettes, examples of which are shown in Fig.A1.4, have three measuring grids arranged at 60° from each other. Taking the axis of grid 1 as a datum, ϵ_1 is measured by grid 1, at α = 0°, ϵ_2 is measured by grid 2, at α = 60°, and ϵ_3 is measured by grid 3, at α = 120°. These measured levels can be substituted into equation A1:5 to give ϵ_x, ϵ_y and their direction referred to arbitary axes, and β_{xy} in terms of the measured levels so that,

$$\text{in plane 1, } \varepsilon_1 = \varepsilon_x \cos^2 0 + \varepsilon_y \sin^2 0 + \beta_{xy} \sin 0 \cos 0 \qquad \text{(A1:10)}$$

$$\text{in plane 2, } \varepsilon_2 = \varepsilon_x \cos^2 60 + \varepsilon_y \sin^2 60 + \beta_{xy} \sin 60 \cos 60 \qquad \text{(A1:11)}$$

$$\text{in plane 3, } \varepsilon_3 = \varepsilon_x \cos^2 120 + \varepsilon_y \sin^2 120 + \beta_{xy} \sin 120 \cos 120 \qquad \text{(A1:12)}$$

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Fig.A1.4. Typical Delta Rosettes

Taking equation A1:10 and solving for ϵ_x.

$$\varepsilon_1 = \varepsilon_x \times 1^2 + \varepsilon_y 0 + \beta_{xy} \times 0 \times 1$$

so that $\varepsilon_1 = \varepsilon_x$

Taking equation A1:11 and resolving for ϵ_y

$$\varepsilon_2 = \varepsilon_x \times \frac{1}{2} \times \frac{1}{2} + \varepsilon_y \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \beta_{xy} \times \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$= \frac{\varepsilon_1}{4} + \frac{3\varepsilon_y}{4} + \frac{\sqrt{3}\beta_{xy}}{4}$$

$$\text{so that } \varepsilon_y = \frac{4}{3}\left(\varepsilon_2 - \frac{\varepsilon_1}{4} + \frac{\sqrt{3}}{4}\beta_{xy}\right)$$

$$= \frac{4\varepsilon_2}{3} - \frac{\varepsilon_1}{3} - \frac{\beta_{xy}}{\sqrt{3}}$$

Now resolving equation A1:12 for ϵ_y

$$\varepsilon_3 = \varepsilon_x \times -\frac{1}{2} \times -\frac{1}{2} + \varepsilon_y \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \beta_{xy} \times \frac{\sqrt{3}}{2} \times -\frac{1}{2}$$

$$= \frac{\varepsilon_1}{4} + \frac{3\varepsilon_y}{4} - \frac{\sqrt{3}}{4}\beta_{xy}$$

$$\text{so that } \varepsilon_y = \frac{4}{3}\left(\varepsilon_3 - \frac{\varepsilon_1}{4} + \frac{\sqrt{3}}{4}\beta_{xy}\right)$$

$$= \frac{4\varepsilon_3}{3} - \frac{\varepsilon_1}{3} + \frac{\beta_{xy}}{\sqrt{3}}$$

Equating the two expressions for ϵ_y.

$$\frac{4\varepsilon_2}{3}-\frac{\varepsilon_1}{3}-\frac{\beta_{xy}}{\sqrt{3}}=\frac{4\varepsilon_3}{3}-\frac{\varepsilon_1}{3}+\frac{\beta_{xy}}{\sqrt{3}}$$

$$\text{and } \frac{4\varepsilon_2}{3}-\frac{\varepsilon_1}{3}-\frac{4\varepsilon_3}{3}+\frac{\varepsilon_1}{3}=\frac{2\beta_{xy}}{\sqrt{3}}$$

$$\text{so that } \beta_{xy}=\frac{4\sqrt{3}\varepsilon_2}{6}-\frac{4\sqrt{3}\varepsilon_3}{6}$$

$$\beta_{xy}=\frac{2(\varepsilon_2-\varepsilon_3)}{\sqrt{3}}$$

Substituting this value into one of the expressions for ϵ_y,

$$\varepsilon_y=\frac{4\varepsilon_3}{3}-\frac{\varepsilon_1}{3}+\frac{2(\varepsilon_2-\varepsilon_3)}{\sqrt{3}\sqrt{3}}$$

$$=\frac{4\varepsilon_3-\varepsilon_1+2\varepsilon_2-2\varepsilon_3}{3}$$

$$\varepsilon_y=\frac{-\varepsilon_1+2\varepsilon_2+2\varepsilon_3}{3}$$

$$\text{so that } \varepsilon_x=\varepsilon_1$$

$$\varepsilon_y=\frac{-\varepsilon_1+2\varepsilon_2+2\varepsilon_3}{3}$$

$$\text{and } \beta_{xy}=\frac{2(\varepsilon_2-\varepsilon_3)}{\sqrt{3}}$$

These three expressions can be substituted in equations A1:8 and A1:9 to yield the maximum and minimum principal strains in terms of ϵ_1, ϵ_2, and ϵ_3.

First using equation A1:8

$$\varepsilon_{max}=\frac{\varepsilon_x+\varepsilon_y}{2}+\frac{\sqrt{(\varepsilon_x-\varepsilon_y)^2+\beta^2_{xy}}}{2}$$

$$=\frac{\varepsilon_1+\left(\frac{-\varepsilon_1+2\varepsilon_2+2\varepsilon_3}{3}\right)}{2}+\frac{\sqrt{\left[\varepsilon_1-\left(\frac{-\varepsilon_1+2\varepsilon_2+2\varepsilon_3}{3}\right)\right]^2+\left[\frac{2(\varepsilon_2-\varepsilon_2)}{\sqrt{3}}\right]^2}}{2}$$

$$=\frac{\varepsilon_1}{2}+\left(-\frac{\varepsilon_1}{6}+\frac{2\varepsilon_2}{6}+\frac{2\varepsilon_3}{6}\right)+\sqrt{\left[\frac{\varepsilon_1}{2}-\left(-\frac{\varepsilon_1}{6}+\frac{2\varepsilon_2}{6}+\frac{2\varepsilon_3}{6}\right)\right]^2+\left[\frac{2(\varepsilon_2-\varepsilon_3)}{2\sqrt{3}}\right]^2}$$

$$=\frac{\varepsilon_1}{3}+\frac{\varepsilon_2}{3}+\frac{\varepsilon_3}{3}+\sqrt{\left[\varepsilon_1-\frac{\varepsilon_1}{3}+\frac{\varepsilon_2}{3}+\frac{\varepsilon_3}{3}\right]^2+\left[\frac{\varepsilon_2-\varepsilon_3}{\sqrt{3}}\right]^2}$$

$$=\frac{\varepsilon_1+\varepsilon_2+\varepsilon_3}{3}+\sqrt{\left[\varepsilon_1-\frac{\varepsilon_1+\varepsilon_2+\varepsilon_3}{3}\right]^2+\left[\frac{\varepsilon_2-\varepsilon_3}{\sqrt{3}}\right]^2} \quad \text{(A1:13)}$$

Similarly $\epsilon_{min}=\frac{\varepsilon_1+\varepsilon_2+\varepsilon_3}{3}-\sqrt{\left[\varepsilon_1-\frac{\varepsilon_1+\varepsilon_2+\varepsilon_3}{3}\right]^2+\left[\frac{\varepsilon_2-\varepsilon_3}{\sqrt{3}}\right]^2}$ **(A1:14)**

And $\tan 2\alpha_p=\frac{\beta_{xy}}{\varepsilon_x-\varepsilon_y}$

$$=\frac{\dfrac{2(\varepsilon_2-\varepsilon_3)}{\sqrt{3}}}{\varepsilon_1-\left(\dfrac{-\varepsilon_1+2\varepsilon_2+2\varepsilon_3}{3}\right)}$$

Which simplifies to

$$=\frac{\dfrac{2(\varepsilon_2-\varepsilon_3)}{\sqrt{3}}}{2\left(\varepsilon_1-\dfrac{\varepsilon_1+\varepsilon_2+\varepsilon_3}{3}\right)}$$

$$=\frac{\dfrac{1}{\sqrt{3}}(\varepsilon_2-\varepsilon_3)}{\varepsilon_1-\dfrac{\varepsilon_1+\varepsilon_2+\varepsilon_3}{3}}$$

So that $\alpha_p=\frac{1}{2}\tan^{-1}\frac{\dfrac{1}{\sqrt{3}}(\varepsilon_2-\varepsilon_3)}{\varepsilon_1-\dfrac{\varepsilon_1+\varepsilon_2+\varepsilon_3}{3}}$ **(A1:15)**

While $\beta_{max}=\varepsilon_{max}-\varepsilon_{min}$ *(ie A1:13–A1:14)*

$$=2\sqrt{\left[\varepsilon_1-\frac{\varepsilon_1+\varepsilon_2+\varepsilon_3}{3}\right]^2+\left[\frac{\varepsilon_2-\varepsilon_3}{\sqrt{3}}\right]^2} \quad \text{(A1:16)}$$

These values can be substituted into equations 2,4 to give maximum and minimum stress levels, so that,

$$\sigma_{max} = E\left[\frac{\varepsilon_1+\varepsilon_2+\varepsilon_3}{3(1-\mu)} + \frac{1}{1+\mu}\sqrt{\left(\varepsilon_1-\frac{\varepsilon_1+\varepsilon_2+\varepsilon_3}{3}\right)^2+\left(\frac{\varepsilon_2-\varepsilon_3}{\sqrt{3}}\right)^2}\right] \quad \text{(A1:17)}$$

$$\sigma_{min} = E\left[\frac{\varepsilon_1+\varepsilon_2+\varepsilon_3}{3(1-\mu)} - \frac{1}{1+\mu}\sqrt{\left(\varepsilon_1-\frac{\varepsilon_1+\varepsilon_2+\varepsilon_3}{3}\right)^2+\left(\frac{\varepsilon_2-\varepsilon_3}{\sqrt{3}}\right)^2}\right] \quad \text{(A1:18)}$$

The basic relationship between the shear stress and the shear strain

Shear Stress τ = Shear Strain β × Modulus of Rigidity G

is similar to the relationship for normal stress and strain ($\sigma = \epsilon E$).

also $$E = 2G(1+\mu) \quad \text{(A1:19)}$$

so rewriting the shear relationship in terms of E,

$$\tau = \frac{E\beta}{2(1+\mu)} \quad \text{(A1:20)}$$

and inserting the value for β_{max}

$$\tau_{max} = \frac{E}{1+\mu}\sqrt{\left(\varepsilon_1-\frac{\varepsilon_1+\varepsilon_2+\varepsilon_3}{3}\right)^2+\left(\frac{(\varepsilon_2-\varepsilon_3)}{\sqrt{3}}\right)^2} \quad \text{(A1:21)}$$

The Rectangular Rosette

Typical examples of rectangular rosettes are shown in Fig.A1.5. A

Fig.A1.5. Typical Rectangular Rosettes

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similar analysis can be made to yield maximum, minimum, and shear strains and stresses, together with the angle that the principal planes make with the datum gauge grid (grid 1 in Fig.A1.5).

$$\varepsilon_{max} = \frac{\varepsilon_1 + \varepsilon_3}{2} + \frac{1}{2}\sqrt{(\varepsilon_1 - \varepsilon_3)^2 + \left[2\varepsilon_2 - (\varepsilon_1 + \varepsilon_3)\right]^2} \qquad \text{(A1:22)}$$

$$\varepsilon_{min} = \frac{\varepsilon_1 + \varepsilon_3}{2} - \frac{1}{2}\sqrt{(\varepsilon_1 - \varepsilon_3)^2 + \left[2\varepsilon_2 - (\varepsilon_1 + \varepsilon_3)\right]^2} \qquad \text{(A1:23)}$$

$$\alpha_p = \frac{1}{2}\tan^{-1}\frac{2\varepsilon_2 - (\varepsilon_1 + \varepsilon_3)}{\varepsilon_1 - \varepsilon_3} \qquad \text{(A1:24)}$$

$$\beta_{max} = \sqrt{(\varepsilon_1 - \varepsilon_3)^2 + \left[2\varepsilon_2 - (\varepsilon_1 + \varepsilon_3)\right]^2} \qquad \text{(A1:25)}$$

$$\sigma_{max} = \frac{E}{2}\left[\frac{\varepsilon_1 + \varepsilon_3}{1-\mu} + \frac{1}{1+\mu}\sqrt{(\varepsilon_1 - \varepsilon_3)^2 + \left[2\varepsilon_2 - (\varepsilon_1 + \varepsilon_3)\right]^2}\right] \qquad \text{(A1:26)}$$

$$\sigma_{min} = \frac{E}{2}\left[\frac{\varepsilon_1 + \varepsilon_3}{1-\mu} - \frac{1}{1+\mu}\sqrt{(\varepsilon_1 - \varepsilon_3)^2 + \left[2\varepsilon_2 - (\varepsilon_1 + \varepsilon_3)\right]^2}\right] \qquad \text{(A1:27)}$$

$$\tau_{max} = \frac{E}{2(1+\mu)}\sqrt{(\varepsilon_1 - \varepsilon_3)^2 + \left[2\varepsilon_2 - (\varepsilon_1 + \varepsilon_3)\right]^2} \qquad \text{(A1:28)}$$

The T-Delta Rosette

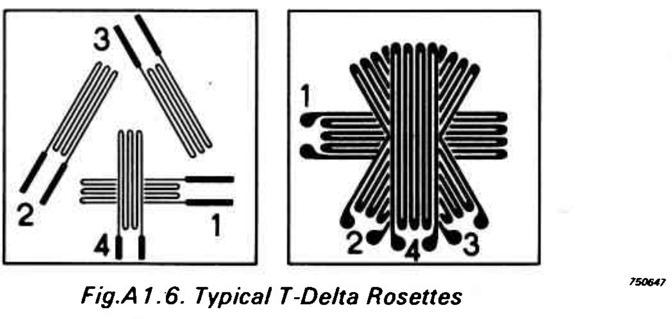


Fig.A1.6. Typical T-Delta Rosettes

The T-delta rosette is basically the same as the delta rosette, but it features a fourth strain gauge grid set at right angles to one of the other grids, see Fig.A1.6. The equations developed for the delta rosette can be used, or the following expressions can be used to take advantage of all four gauge grids.

$$\varepsilon_{max} = \frac{\varepsilon_1 + \varepsilon_4}{2} + \frac{1}{2}\sqrt{(\varepsilon_1 - \varepsilon_4)^2 + \frac{4}{3}(\varepsilon_2 - \varepsilon_3)^2} \tag{A1:29}$$

$$\varepsilon_{min} = \frac{\varepsilon_1 + \varepsilon_4}{2} - \frac{1}{2}\sqrt{(\varepsilon_1 - \varepsilon_4)^2 + \frac{4}{3}(\varepsilon_2 - \varepsilon_3)^2} \tag{A1:30}$$

$$\alpha_p = \frac{1}{2}\tan^{-1}\frac{2(\varepsilon_2 - \varepsilon_3)}{\sqrt{3}(\varepsilon_1 - \varepsilon_4)} \tag{A1:31}$$

$$\beta_{max} = \sqrt{(\varepsilon_1 - \varepsilon_4)^2 + \frac{4}{3}(\varepsilon_2 - \varepsilon_3)^2} \tag{A1:32}$$

$$\sigma_{max} = \frac{E}{2}\left[\frac{\varepsilon_1 + \varepsilon_4}{1-\mu} + \frac{1}{1+\mu}\sqrt{(\varepsilon_1 - \varepsilon_4)^2 + \frac{4}{3}(\varepsilon_2 - \varepsilon_3)^2}\right] \tag{A1:33}$$

$$\sigma_{min} = \frac{E}{2}\left[\frac{\varepsilon_1 + \varepsilon_4}{1-\mu} - \frac{1}{1+\mu}\sqrt{(\varepsilon_1 - \varepsilon_4)^2 + \frac{4}{3}(\varepsilon_2 - \varepsilon_3)^2}\right] \tag{A1:34}$$

$$\tau_{max} = \frac{E}{2(1+\mu)}\sqrt{(\varepsilon_1 - \varepsilon_4)^2 + \frac{4}{3}(\varepsilon_2 - \varepsilon_3)^2} \tag{A1:35}$$

APPENDIX 2

Practical examples showing the use of the equations developed in Appendix 1, and the construction and explanation of Mohr's strain circle

Fig.A2.1 shows a group of three strain gauges arranged to measure strain at three different angles about a point in a strain field. Gauge 1 measures a strain of $+850\mu\epsilon$ (strain being a dimensionless ratio of length to length, the units could equally well be μm/m or μin/in). Gauge 2 measures $-100\mu\epsilon$, and Gauge 3 measures $+350\mu\epsilon$.

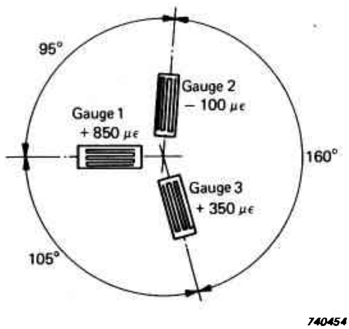


Fig.A2.1. Strain gauge arrangement for the worked example

Calculation method

Taking the axis through Gauge 1 as the X axis, principal strains will be calculated from equation 2:5,

$$\varepsilon_a = \varepsilon_x cos^2 a + \varepsilon_y sin^2 a + \beta_{xy} sin a cos a$$

the position of the principal planes from equation 2:6,

$$tan 2a_p = \frac{\beta_{xy}}{\varepsilon_x - \varepsilon_y}$$

and the principal stresses from equations 2:4,

$$\sigma_x = \frac{E}{1-\mu^2}(\varepsilon_x + \mu\varepsilon_y)$$

$$\sigma_y = \frac{E}{1-\mu^2}(\varepsilon_y + \mu\varepsilon_x)$$

First, using the values obtained from Gauge 1 in Equation 2:5,

$$850 = \varepsilon_x \cos^2 0 + \varepsilon_y \sin^2 0 + \beta_{xy} \sin 0 \cos 0$$

$$\varepsilon_x = 850\ \mu\varepsilon$$

Now using the values from Gauge 2,

$$-100 = 850(-0{,}0872)^2 + \varepsilon_y(0{,}9962)^2 + \beta_{xy}(0{,}9962)(-0{,}0872)$$

$$-100 = 6{,}463 + 0{,}9926\ \varepsilon_y - 0{,}8688\ \beta_{xy}$$

$$\varepsilon_y = \frac{-106{,}463 + 0{,}08688\ \beta_{xy}}{0{,}9926}$$

$$\varepsilon_y = -107{,}2 + 0{,}08752\ \beta_{xy}$$

Using the values from Gauge 3,

$$350 = 850(-0{,}2588)^2 + \varepsilon_y(-0{,}9659)^2 + \beta_{xy}(-0{,}9659)(-0{,}2588)$$

$$350 = 56{,}94 + 0{,}9328\ \varepsilon_y + 0{,}250\ \beta_{xy}$$

$$\varepsilon_y = \frac{293{,}06 - 0{,}250\ \beta_{XY}}{0{,}9328}$$

$$\varepsilon_y = 314{,}2 - 0{,}268\ \beta_{xy}$$

Equating the two values obtained for ϵ_y

$$-107{,}2 + 0{,}08752\ \beta_{xy} = 314{,}2 - 0{,}268\ \beta_{xy}$$

$$(0{,}08752 + 0{,}268)\ \beta_{xy} = 314{,}2 + 107{,}2$$

$$\beta_{xy} = \frac{421{,}4}{0{,}3555}$$

$$\beta_{xy} = 1185\ \mu\varepsilon$$

(This is actually equivalent to twice the height to the intercept of the Mohr strain circle and ϵ_1 axis in Fig.A2.3, as this axis is the datum for the calculations.)

$$\varepsilon_y = -107{,}2 + 0{,}08752 \times 1185$$

Therefore $\quad \varepsilon_y = -107{,}2 + 103{,}8$

$$\varepsilon_y = -3{,}4\ \mu\varepsilon$$

Hence the solution for ϵ_x, ϵ_y, and β_{xy} referred to the datum axis:

$$\varepsilon_x = 850\,\mu\varepsilon \quad : \quad \varepsilon_y = -3{,}4\,\mu\varepsilon \quad : \quad \beta_{xy} = 1185\,\mu\varepsilon$$

Therefore $$tan2\alpha_p = \frac{1185}{850-(-3{,}4)} \qquad (2:6)$$

$$tan2\alpha_p = \frac{1185}{853{,}4}$$

$$tan2\alpha_p = 1{,}39$$

So $$2\alpha_p = 54{,}3^\circ \text{ or } 234{,}3^\circ$$

(Because the tangent is the ratio of two positive numbers, had they been two negatives, 234,3° would have been the correct solution.)

α_p = 27,15° from Gauge 1 datum plane

Now putting α_p back into the original equations,

$$\varepsilon_{max} = \varepsilon_x cos^2\alpha_p + \varepsilon_y sin^2\alpha_p + \beta_{xy} sin\alpha_p cos\alpha_p$$

$$\varepsilon_{max} = 850cos^2 27{,}15^\circ + (-3{,}4)sin^2 27{,}15^\circ + 1185sin27{,}15^\circ cos27{,}15^\circ$$

$$\varepsilon_{max} = 850(0{,}890)^2 - 3{,}4(0{,}450)^2 + 1185(0{,}450)(0{,}890)$$

$$\varepsilon_{max} = 673{,}3 - 0{,}69 + 474{,}7$$

$$\boldsymbol{\varepsilon_{max} = 1147{,}3}\,\mu\varepsilon$$

ϵ_{min} occurs at 90° from ϵ_{max} so solving for ϵ_{min} = (α_p ± 90°)

$$\varepsilon_{min} = 850cos^2 117{,}15^\circ + (-3{,}4)sin^2 117{,}15^\circ + 1185sin117{,}15^\circ cos117{,}15^\circ$$

$$\varepsilon_{min} = 850(-0{,}450)^2 - 3{,}4(0{,}890)^2 + 1185(0{,}890)(-0{,}450)$$

$$\varepsilon_{min} = 172{,}1 - 2{,}7 - 474{,}7$$

$$\boldsymbol{\varepsilon_{min} = -305{,}3}\,\mu\varepsilon$$

And $$\beta = \varepsilon_{max} - \varepsilon_{min}$$

$$\beta = 1147{,}3 - (-305{,}3)$$

$$\beta = \mathbf{1452{,}6}\,\mu\varepsilon$$

If the three strain gauges in the example had been fastened on to an aircraft structure made of aluminium with Young's Modulus E = 70 GPa (7000 kp/mm² = 107 lbf/in²) and Poissons Ratio μ = 0,32, these and the measured values could be substituted in equations 2:4 to yield the Principal Stresses.

$$\sigma_{max} = \frac{E}{1-\mu^2}\left(\varepsilon_{max} + \mu\varepsilon_{min}\right)$$

$$\sigma_{min} = \frac{E}{1-\mu^2}\left(\varepsilon_{min} + \mu\varepsilon_{max}\right)$$

First in ISO Units:

$$\varepsilon_{max} = \frac{70}{1-0{,}32^2}\left(1147{,}3-0{,}32\times 305{,}3\right)10^{-6}\ GPa$$

$$= \frac{70\times 1049{,}6\times 10^{-6}}{0{,}8976}$$

$$= 81{,}85\ MPa\ (8{,}185\ kp/mm^2)$$

and $$\sigma_{min}{}^2 = \frac{70}{1-0{,}32^2}\left(-305{,}3+0{,}32\times 1147{,}3\right)10^{-6}\ GPa$$

$$= \frac{70\times 61{,}83\times 10^{-6}}{0{,}8976}$$

$$= 4{,}82\ MPa\ (0{,}482\ kp/mm^2)$$

Similarly in British Units:

$$\sigma_{max} = \frac{10^7}{1-0{,}32^2}\left(1147{,}3-0{,}32\times 305{,}3\right)10^{-6}\ lb/in^2$$

$$= \frac{10^7\times 1049{,}6\times 10^{-6}}{0{,}8976}$$

$$= 11693\ lb/in^2$$

and $$\sigma_{min} = \frac{10^7}{1-0{,}32^2}\left(-305{,}3+0{,}32\times 1147{,}3\right)10^{-6}\ lb/in^2$$

$$= \frac{10^7\times 61{,}83\times 10^{-6}}{0{,}8976}$$

$$= 688\ lb/in^2$$

The shear stress can be calculated from

$$\tau = \frac{E\beta}{2(1+0{,}32)} \tag{A1:20}$$

ISO Units: $$\tau = \frac{70 \times 1452{,}6 \times 10^{-6}}{2(1+0{,}32)} \quad GPa$$

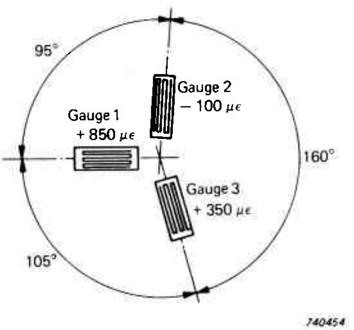
$$= 38{,}5 \; MPa \; (3{,}85 \; kp/mm^2)$$

British Units: $$\tau = \frac{10^7 \times 1452{,}6 \times 10^{-6}}{2(1+0{,}32)}$$

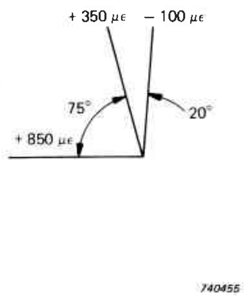
$$= 5502 \; lb/in^2$$

Graphical method using Mohr's strain circle

There is a graphical method, devised by Mohr, that keeps track of the relationship between strains and their directions at a point in the strain field. It is easiest to demonstrate Mohr's Strain Circle by means of an actual worked example. Fig.A2.2a shows the three strain gauges arranged to measure strain at three different angles about a point in a strain field as in the previous example. Gauge 1 measures $+850\mu\epsilon$, Gauge 2 measures $-100\mu\epsilon$, and Gauge 3 measures $+350\mu\epsilon$. Construction of the Strain Circle is realised in the following series of steps.



A2.2a. Strain gauge arrangement



A2.2b. Rearrangement of axes

Fig.A2.2. Worked example for Mohr's strain circle

1. **Rearrangement of geometry** where necessary, to get the axes of measurement of all three gauges within a total included angle less than 180°, and to place the axis line of the gauge with intermediate strain magnitude between the axes of the other two gauges. This is only a geometrical exercise, and it should be emphasised that it does **not** involve actually moving the gauges. Fig.A2.2b shows this rearrangement of the axis lines.

2. **Draw a vertical axis** (shown as $\beta/2$ in Fig.A2.3), and with positive values to the right, draw a series of parallel vertical axes at distances from the first axis that correspond to the strains measured by each of the gauges. These axes are shown as ϵ_1, ϵ_2 and ϵ_3.

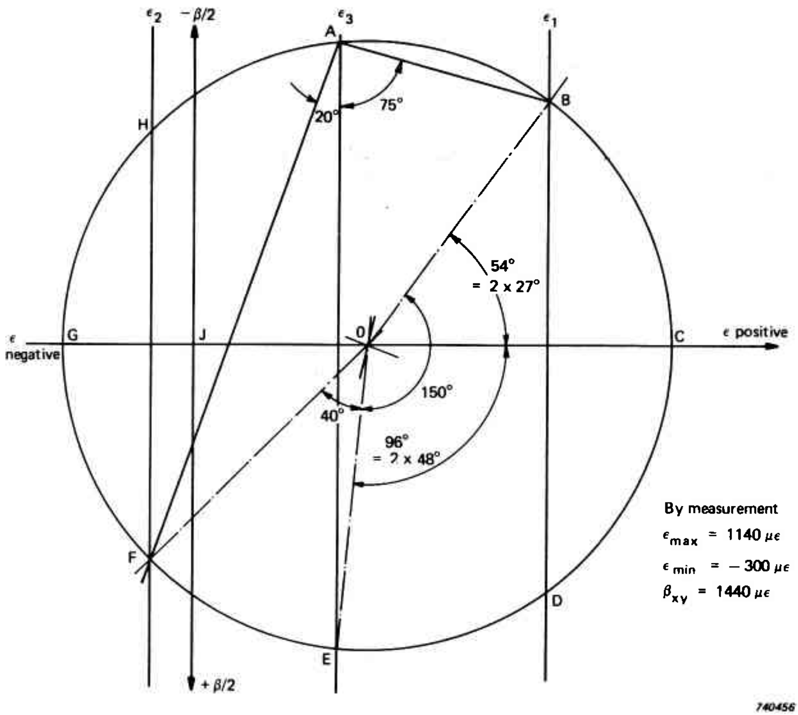


Fig.A2.3. Mohr's Strain Circle for the worked example

3. **Draw the same angle** that the axis having the smallest strain makes with the axis having intermediate-strain in the rearranged geometry diagram (20°) clockwise from an arbitary point A on the axis corresponding to the intermediate strain value (ϵ_3). Continue the angle until it crosses the axis for smallest-strain (ϵ_2) at point F.

4. **Similarly draw the angle** that the axis having the largest-strain makes with the axis having intermediate-strain in the rearranged geometry diagram (75°) from the same point A on the other side of the axis for the intermediate strain value (ϵ_3). Continue the angle until the line crosses the axis for largest-strain (ϵ_1) at point B.

5. **Construct a circle** passing through points A, B and F with its centre O where the bisectors of A — B and A — F cross.

6. **Draw a horizontal axis** through the centre of the circle O.

7. **Measure** ϵ_{max} which is given by the distance J — C.

8. **Measure** ϵ_{min} which is given by the distance G — J.

9. **Measure** β which is given by G — C the diameter of the circle = max — min. (strictly $\beta = 2 \times$ the radius of the circle.)

10. **Find the angle** of the principal planes. Starting at one of the points where the circle crosses the smallest-strain axis (ϵ_2), at point H or F, move counterclockwise around the circumference of the circle through 2 × the angle between the smallest-strain axis and the intermediate strain axis shown in the rearranged geometry diagram until one of the points where the circle crosses the intermediate axis (ϵ_3) at points A or E. In the example shown here, the only angle that can satisfy the requirement is FOE = 2 × 20° = 40°. Move on round the circumference from point E through 2 × the angle that the largest-strain axis makes with the intermediate strain axis in the rearranged geometry diagram. The new point should also be where the circle crosses the largest-strain axis (ϵ_1); it is at point B. The two angles FOE and EOB represent the angles of the rearranged axes in Fig.A2.2b while the horizontal axis through the centre of the circle represents the direction of the principal planes. One principal plane (the maximum O — C) lies between the intermediate and the largest-strain measured axes, making an angle EOC with the line representing the intermediate-strain axis, and an angle COB with the line representing the largest-

strain axis in the rearranged diagram. Therefore, in the rearranged geometry diagram, the maximum principal plane lies at an angle EOC/2 from the intermediate axis, and at an angle BOC/2 from the largest-strain axis. The other principal plane (the minimum) lies at 90° to the first. Fig.A2.4a shows the rearranged geometry diagram with the principal axes drawn in, and it is a comparatively simple matter to extrapolate back to the orginal layout to find where the principal planes lie. This is shown in Fig.A2.4b.

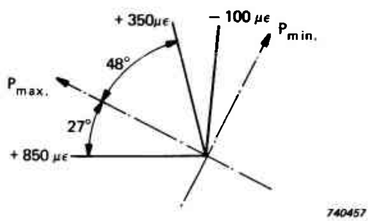


Fig.A2.4a. *Principal planes on rearranged axis diagram*

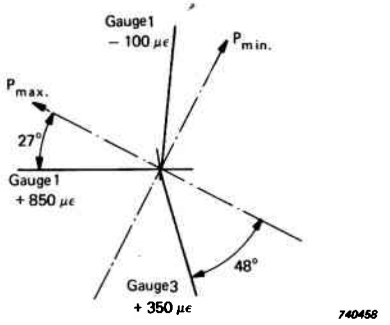


Fig.A2.4b. *Principal planes drawn on the original layout*

Fig.A2.4. Position of the principal planes

The final results of the worked example, as measured from the Mohr Strain Circle are as follows:

$$\varepsilon_{max} = +1140\mu\varepsilon$$
$$\varepsilon_{min} = -300\mu\varepsilon$$
$$\beta_{xy} = +1440\mu\varepsilon$$

and $\alpha_p = 27°$ measured from the Gauge 1 line.

These results can be compared with the results obtained by the calculations shown previously.

Now having obtained results, how does the method work? Reasoning onwards from the construction employed, the convention used for the axes is Normal Stress (ϵ) horizontal with positive values to the right and negative values to the left. Shear Stress (β) graduated in units of $\beta/2$, is drawn vertically with positive values downwards. This suits β_{xy} in equation (A1:5 = 2:5), β_{yx} is of equal magnitude but opposite sign. The sign convention and arrangement adopted permit one single point on the diagram to represent the state of strain on some plane (X in Fig.A2.5) in a strain field, so that all points on the circumference of the circle represent strains on all the possible planes.

In equations A1:6 and A1:7 (= 2:6) the angles given are two times the angle between the datum plane and the plane being calculated. Using the theorem that says that the angle subtended at the centre of a circle is twice the angle subtended at the circumference, the strain circle is so constructed to yield these double angles at the centre. Therefore the principal planes are at 180° from each other where the circle crosses the Normal Strain axis, that is, where the Shearing Strain is zero.

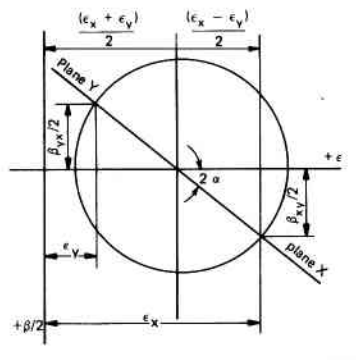


Fig.A2.5. Geometry of Mohr's Strain Circle

Further examination of the diagram in Fig.A2.5, shows that the right-angle triangle that contains the angle 2α has a base length

$$= \varepsilon_x - \frac{\varepsilon_x + \varepsilon_y}{2}$$

$$= \frac{\varepsilon_x - \varepsilon_y}{2}$$

and height $= \beta_{xy}/2$

so that $\tan 2\alpha = \frac{\beta_{xy}}{2} / \frac{\varepsilon_x - \varepsilon_y}{2}$

$$\tan 2\alpha = \frac{\beta_{xy}}{(\varepsilon_x - \varepsilon_y)}$$

which is equation A1:7 (= 2:6)

The hypotenuse of the triangle

$$=\sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\beta_{xy}}{2}\right)^2}$$

so that $$\sin 2a=\frac{\beta_{xy}}{2}\,\frac{1}{\sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\beta_{xy}}{2}\right)^2}}$$

and $$\cos 2a=\frac{\varepsilon_x-\varepsilon_y}{2}\,\frac{1}{\sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\beta_{xy}}{2}\right)^2}}$$

From the method given for the worked example, the magnitude of the maximum Normal Strain is given by J — C, which is equivalent to the distance from the vertical axis to the centre of the circle plus the radius (hypotenuse) of the circle.

that is $$\varepsilon_a=\frac{\varepsilon_x+\varepsilon_y}{2}+\sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\beta_{xy}}{2}\right)^2}$$

$$=\frac{\varepsilon_x+\varepsilon_y}{2}+\frac{\sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\beta_{xy}}{2}\right)^2}}{1}\times\frac{\sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\beta_{xy}}{2}\right)^2}}{\sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\beta_{xy}}{2}\right)^2}}$$

$$=\frac{\varepsilon_x+\varepsilon_y}{2}+\frac{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2}{\sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\beta_{xy}}{2}\right)^2}}+\frac{\left(\frac{\beta_{xy}}{2}\right)^2}{\sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\beta_{xy}}{2}\right)^2}}$$

$$\varepsilon_a=\frac{\varepsilon_x+\varepsilon_y}{2}+\frac{\varepsilon_x-\varepsilon_y}{2}\cos 2a+\frac{\beta_{xy}}{2}\sin 2a$$

which is equation A1:6.

Examining Fig.A2.5 once again to find where the minimum Normal Strain occurs, another right-angle triangle containing $2a$ is found. This triangle is in the third quadrant of our sign convention so that

$$\sin 2a=\frac{-\beta_{xy}}{2}\,\frac{1}{\sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\beta_{xy}}{2}\right)^2}}\qquad(\beta_{yx}=-\beta_{xy})$$

$$cos2a = \frac{-(\varepsilon_x - \varepsilon_y)}{2} \frac{1}{\sqrt{(\frac{\varepsilon_x - \varepsilon_y}{2})^2 + (\frac{\beta_{xy}}{2})^2}}$$

From equation A1:6 it can be deduced that the radius of the strain circle

$$= \frac{\varepsilon_x - \varepsilon_y}{2} cos2a + \frac{\beta_{xy}}{2} sin2a$$

but here in the third quadrant

$$= -\frac{\varepsilon_x - \varepsilon_y}{2} cos2a - \frac{\beta_{xy}}{2} sin2a$$

Hence the minimum Normal Strain is given by the distance from the vertical axis to the centre of the circle minus the circle radius as was measured for the worked example (G — J). This still obeys equation A1:6 as the terms are to be added algebraically.

Now it can be noted that the Mohr Strain Circle is a geometrical construction specially designed to contain elements that satisfy the equations used for finding principal strains. It can be employed to find the position of the principal planes and hence the maximum and minimum Normal Strains, and the Shearing Strain present at a point in a strain field when the strain magnitude in three directions is known. The accuracy of the final results depends greatly upon the accuracy and skill of the draftsman constructing the diagram. Therefore it is suggested that the circle be sketched freehand, and used purely as a reminder of the geometrical relationships between the strain components, so that the various strains can be calculated on a basis of these geometrical relationships.

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PRINTED IN DENMARK · K. LARSEN & SØN A/S · DK-2860 SØBORG