Journal of Sound and Vibration ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Force transmissibility versus displacement transmissibility

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ARTICLE INFO

Article history:
Received 5 June 2013
Received in revised form
18 May 2014
Accepted 19 May 2014
Handling Editor: H. Ouyang

ABSTRACT

It is well-known that when a single-degree-of-freedom (sdof) system is excited by a continuous motion of the foundation, the force transmissibility, relating the force transmitted to the foundation to the applied force, equals the displacement transmissibility. Recent developments in the generalization of the transmissibility to multiple-degree-of-freedom (mdof) systems have shown that similar simple and direct relations between both types of transmissibility do not appear naturally from the definitions, as happens in the sdof case.

In this paper, the authors present their studies on the conditions under which it is possible to establish a relation between force transmissibility and displacement transmissibility for mdof systems. As far as the authors are aware, such a relation is not currently found in the literature, which is justified by being based on recent developments in the transmissibility concept for mdof systems. Indeed, it does not appear naturally, but the authors observed that the needed link is present when the displacement transmissibility is obtained between the same coordinates where the applied and reaction forces are considered in the force transmissibility case; this implies that the boundary conditions are not exactly the same and instead follow some rules.

This work presents a formal derivation of the explicit relation between the force and displacement transmissibilities for mdof systems, and discusses its potential and limitations. The authors show that it is possible to obtain the displacement transmissibility from measured forces, and the force transmissibility from measured displacements, opening new perspectives, for example, in the identification of applied or transmitted forces. With this novel relation, it becomes possible, for example, to estimate the force transmissibility matrix with the structure off its supports, in free boundary conditions, and without measuring the forces. As far as force identification is concerned, this novel approach significantly decreases the computational effort when compared to conventional approaches, as it requires only local information of the sets of coordinates involved. Numerical simulations and experimental examples are presented and discussed, to illustrate the proposed developments.

1. Introduction

The displacement transmissibility in an sdof system is defined as the ratio between the amplitude of the response displacement and the amplitude of the displacement imposed at the foundation (e.g., [1]). Similarly, the dynamic transmissibility of forces is defined as the ratio between the amplitude of the force transmitted to the ground and the

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http://dx.doi.org/10.1016/j.jsv.2014.05.038

amplitude of the excitation force. In the sdof case, the conclusion is that the transmissibility expressions for both displacement and force are identical.

Although sdof models can be applied to a variety of vibration problems, in many situations the need for higher-order models is evident. However, from its nature and as explained in [2], transmissibility expressions for both force and displacement are not identical for mdof systems. Until now, no answer to the problem of relating these force and displacement mdof transmissibilities has been published, as far as the authors are aware. This fact has motivated the authors to develop their efforts and establish such a relationship.

Before going into the subject of relating displacement and force transmissibility, one must go back a little in time. The problem of extending the idea of displacement transmissibility to an mdof system is essentially the question of how to relate a set of unknown responses to a set of known responses, for a given set of applied forces. On the other hand, extending the idea of force transmissibility to an mdof system is essentially a problem of how to relate a set of reaction forces to a set of applied ones. Some initial attempts are due to Snowdon [3], Vakakis et al. [4–6], Liu et al. [7,8], Varoto et al. [9], and Sciulli et al. [10]. Similar efforts can also be found in the indirect measurement of vibration excitation forces [11–15]. A more general approach has been pursued since the late 1990s [16–19] for the displacement transmissibility as well as for the force transmissibility [20].

Various applications of transmissibility approach may now be found, such as structural response estimation [21], damage detection [22,23], operational modal analysis [24], evaluation of unmeasured frequency response functions (FRFs) [25,26], and force identification [27,28]. Another example is the problem of transfer-path analysis in vibroacoustics, where classical techniques based on measured transfer functions and estimated source strengths may often be time consuming and error prone. Tcherniak et al. [29–31] sought easier and more reliable ways to address those types of problem through the use of the transmissibility matrix extracted from operating measurements and therefore not requiring the measurement of transfer functions.

A review of the multiple applications of the transmissibility concept has been published recently in [2]. The presented concept significantly decreases the computational effort when compared to conventional approaches as it requires only local information of the sets of coordinates involved.

The definition of a relation between force and displacement transmissibilities is the objective of this article and its main novelty. In this article, the authors propose a potentially useful application of this new definition: to estimate the force transmissibility without requiring the structure to be in its operational conditions. Instead, the force transmissibility is obtained from the displacement transmissibility, with the structure in free boundary conditions.

In Sections 2.1 and 2.2, the authors briefly review the theoretical development of the mdof force transmissibility and of the mdof displacement transmissibility, proposed in [18,20]. In Section 3, it is observed that a relationship between them is possible, although subject to some restrictions. The authors have developed and implemented numerical and experimental tests, which are described in Section 4 and illustrated in Section 5, through the presentation of several examples to assess and validate the proposed relationship and methodologies.

2. Transmissibility concepts in mdof systems

In the next two sub-sections, the main definitions of the generalized transmissibility concepts are revisited.

2.1. Force transmissibility in mdof systems

For the introduction to the generalized force transmissibility, the definition of some sets of coordinates is essential. Let us assume that (i) K is a set defined by the coordinates where the external forces are applied, (ii) U is a set defined by the coordinates where the reaction forces appear due to displacement constraints at the supports, and finally (iii) all the

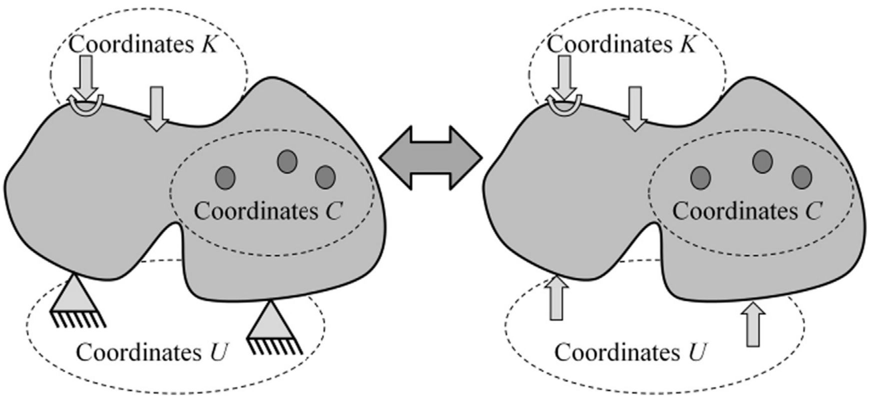

Fig. 1. Illustration of the sets of coordinates K, U and C.

remaining coordinates constitute the set C. Note that in spite of the fact that here all the forces are known, the notation U that was introduced for "unknown" in [20] is kept for the coordinates associated to the given displacement boundary conditions. In that context, K stands for "known". A schematic illustration of the different kind of coordinates is presented in Fig. 1.

The receptance frequency response matrix $\mathbf{H}$ relates, in steady-state conditions, the dynamic displacement amplitudes $\mathbf{Y}$ with the external force amplitudes $\mathbf{F}$. This matrix corresponds to the body free in space (i.e., with all restrictions in subset U removed). The displacement responses $\mathbf{Y}$ at the discretized nodes of the structure (associated to the previously introduced sets K, U and C) may be defined as in the following expression:

$$\begin{Bmatrix} \mathbf{Y}_K \\ \mathbf{Y}_U \\ \mathbf{Y}_C \end{Bmatrix} = \begin{bmatrix} \mathbf{H}_{KK} & \mathbf{H}_{KU} \\ \mathbf{H}_{UK} & \mathbf{H}_{UU} \\ \mathbf{H}_{CK} & \mathbf{H}_{CU} \end{bmatrix} \begin{Bmatrix} \mathbf{F}_K \\ \mathbf{F}_U \end{Bmatrix} \tag{1}$$

Supposing that at the set U the supports totally constraint the displacements, one may impose $\mathbf{Y}_U = \mathbf{0}$; thus,

$$\mathbf{H}_{UK}\ \mathbf{F}_K + \mathbf{H}_{UU}\ \mathbf{F}_U = \mathbf{0} \tag{2}$$

from which,

$$\mathbf{F}_U = \mathbf{T}_{UK}^{(f)} \mathbf{F}_K \tag{3}$$

where

$$\mathbf{T}_{UK}^{(f)} = -(\mathbf{H}_{UU})^{-1} \mathbf{H}_{UK} \tag{4}$$

is the force transmissibility between the sets U and K. The upper index (f) denotes force transmissibility.

The inverse relation, i.e., between the sets U and K, also results from expression (2) and is given by

$$\mathbf{F}_K = (\mathbf{T}_{UK}^{(f)})^{+} \mathbf{F}_U \tag{5}$$

where $(\mathbf{T}_{UK}^{(f)})^{+}$ is the pseudo-inverse of $\mathbf{T}_{UK}^{(f)}$:

$$(\mathbf{T}_{UK}^{(f)})^{+} = -(\mathbf{H}_{UK})^{+} \mathbf{H}_{UU} \tag{6}$$

While in the transmissibility of forces between U and K, in Eq. (4), one has the inversion of a square matrix, in the inverse relation in Eq. (6), a pseudo-inversion (in general) is required, where $\#U$ the number of coordinates in U has to be greater or equal to the number of coordinates $\#K$ of K.

A generalization to the case of elastic supports is considered in the next lines.

When non-zero support displacements are involved, i.e., $\mathbf{Y}_U \neq \mathbf{0}$, one can write the following relation (from Eq. (1)):

$$\mathbf{F}_U = (\mathbf{H}_{UU})^{-1} (\mathbf{Y}_U - \mathbf{H}_{UK}\ \mathbf{F}_K) \tag{7}$$

Considering Eq. (4), one obtains

$$\mathbf{F}_U = \mathbf{T}_{UK}^{(f)} \mathbf{F}_K + (\mathbf{H}_{UU})^{-1} \mathbf{Y}_U \tag{8}$$

For the sake of simplicity, the present article is limited to the case of rigid supports.

2.2. *Displacement transmissibility in mdof systems*

For the introduction of the generalized displacement transmissibility, the definition of some sets of coordinates is also essential. Keeping the designations introduced in [18], let us assume that (i) K is defined as the set of coordinates where the responses $\mathbf{Y}$ are known, (ii) U is defined as the set of coordinates where the responses $\mathbf{Y}$ are unknown, (iii) A is defined as the set of coordinates where forces $\mathbf{F}$ are applied and (iv) all other coordinates of the elastic body discretization constitute the set C. A schematic illustration of this is presented in Fig. 2.

With these sets, one may relate the responses with the excitation forces in all coordinates by the following expression:

$$\begin{Bmatrix} \mathbf{Y}_A \\ \mathbf{Y}_U \\ \mathbf{Y}_K \\ \mathbf{Y}_C \end{Bmatrix} = \begin{bmatrix} \mathbf{H}_{AA} \\ \mathbf{H}_{UA} \\ \mathbf{H}_{KA} \\ \mathbf{H}_{CA} \end{bmatrix} \{\mathbf{F}_A\} \tag{9}$$

To analyze the displacement transmissibility, one is interested in the case where the structure is considered free of its supports. This means that in the model for displacement transmissibility no coordinates have supports where reactions can develop. Due to this, the only non-zero forces are in the set A.

Based on harmonically applied forces at coordinates A, one may establish that displacements at coordinates U and K are related to the applied forces at coordinates A by the following relationships:

$$\mathbf{Y}_U = \mathbf{H}_{UA} \mathbf{F}_A \tag{10}$$

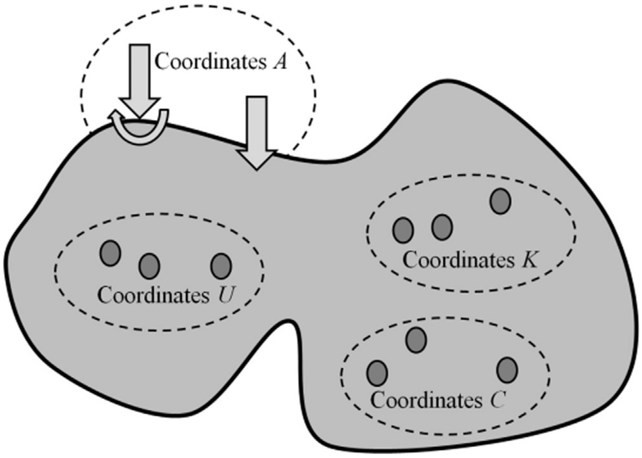

Fig. 2. Illustration of a free elastic body with the four sets of coordinates K, U, C and A.

$$\mathbf{Y}_K = \mathbf{H}_{KA}\mathbf{F}_A \tag{11}$$

Eliminating the external forces $\mathbf{F}_A$ between (10) and (11), one obtains

$$\mathbf{Y}_U = \mathbf{T}_{UK}^{(d)}\mathbf{Y}_K \tag{12}$$

where

$$\mathbf{T}_{UK}^{(d)} = \mathbf{H}_{UA}(\mathbf{H}_{KA})^+ \tag{13}$$

is the transmissibility matrix relating both sets of displacements. $(\mathbf{H}_{KA})^+$ is the pseudo-inverse of the sub-matrix $\mathbf{H}_{KA}$. One important aspect behind this definition is that sub-matrices $\mathbf{H}_{UA}$ and $\mathbf{H}_{KA}$ may be obtained experimentally. The upper index (d) denotes displacement transmissibility, which is implicitly related to the set A.

Note that relation (13) depends not only on the set of coordinates U and K, but also on the set A. Obviously, an inverse relation can also be obtained for $\mathbf{T}_{UK}^{(d)}$ from expressions (10) and (11).

3. Relation between force and displacement transmissibilities

Start by recalling the displacement transmissibility given by Eq. (13), with the structure in free boundary conditions as in Fig. 2. Since there are no restrictions on how the set A is constructed, one may assume that sets A and U coincide, i.e., one can relate $\mathbf{Y}_U$ and $\mathbf{Y}_K$ when the applied forces act at the coordinates U. In such a situation, the displacement transmissibility becomes:

$$\mathbf{T}_{UK}^{(d)} = \mathbf{H}_{UU}(\mathbf{H}_{KU})^+ \tag{14}$$

Now, recalling the force transmissibility given by Eq. (4), one can relate Eqs. (4) and (14). The pseudo-inverse of Eq. (14) is

$$(\mathbf{T}_{UK}^{(d)})^+ = \mathbf{H}_{KU}(\mathbf{H}_{UU})^{-1} \tag{15}$$

Multiplying (15) by –1 and transposing, yields

$$-((\mathbf{T}_{UK}^{(d)})^+)^{\mathrm{T}} = -(\mathbf{H}_{UU})^{-1}\mathbf{H}_{UK} \tag{16}$$

Comparing Eqs. (16) and (4), it follows that

$$\mathbf{T}_{UK}^{(f)} = -((\mathbf{T}_{UK}^{(d)})^+)^{\mathrm{T}} \tag{17}$$

where $\#U \geq \#K$. Conversely, one may establish the following:

$$\mathbf{T}_{UK}^{(d)} = -((\mathbf{T}_{UK}^{(f)})^{\mathrm{T}})^+ \tag{18}$$

where $\#K \geq \#U$.

Eq. (17) allows us to obtain the force transmissibility between two sets of coordinates (the set U, associated with the supports of the structure where the reactions appear, and the set K, associated with the points where the external loads are applied) from the displacement transmissibility between the same two sets of coordinates (although in this case the measurements are taken with the structure free in space).

One can observe that Eqs. (17) and (18) present some limitations for the direct application, but the great advantage of those relations and their possible applications come from the fact that it is possible to obtain the force transmissibility for the mdof case without measuring forces; in fact, they only require the measurement of displacements.

It should also be noted that, whenever possible, the transmissibility matrices should be calculated from the measured receptance FRFs (Eqs. (4) and (13)), taking into account the limitations of each formulation.

3.1. Verification for the sdof case

It seems appropriate to verify expressions (17) and (18) for the sdof case. Let us consider the sdof system illustrated in Fig. 3. The transmissibility of forces is defined as the ratio between the amplitudes of the transmitted and the applied forces, and it is well-known from any mechanical vibration text book that the result is given by

$$T_{UK}^{(f)} = \frac{\overline{F}_U}{F_K} = \frac{k + \mathrm{i}\omega c}{k - \omega^2 m + \mathrm{i}\omega c} \tag{19}$$

where ω is the excitation frequency, k represents the constant stiffness, m the constant mass and c the damping coefficient of an ideal massless viscous damper, or, simply, by its amplitude:

$$T_{UK}^{(f)} = \frac{F_U}{F_K} = \sqrt{\frac{k^2 + (\omega c)^2}{(k - \omega^2 m)^2 + (\omega c)^2}} \tag{20}$$

In order to verify expressions (4) and (14), one must refer to Fig. 4, which represents the system of Fig. 3, but free in space. One defines the displacement transmissibility by relating the amplitude of the response, $(\overline{Y}_K)$, to the imposed displacement amplitude at the base, (Y_U).

The dynamic equilibrium equations for the free vibration case of Fig. 4 are given by the following expressions:

$$\begin{aligned} m\ddot{y}_K + c(\dot{y}_K - \dot{y}_U) + k(y_K - y_U) &= 0 \\ c(\dot{y}_U - \dot{y}_K) + k(y_U - y_K) &= 0 \end{aligned} \tag{21}$$

As $y_U = Y_U e^{\mathrm{i}\omega t}$ and $y_K = \overline{Y}_K e^{\mathrm{i}\omega t}$, Eq. (21) become:

$$\begin{bmatrix} k - \omega^2 m + \mathrm{i}\omega c & -k - \mathrm{i}\omega c \\ -k - \mathrm{i}\omega c & k + \mathrm{i}\omega c \end{bmatrix} \begin{Bmatrix} \overline{Y}_K \\ Y_U \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{22}$$

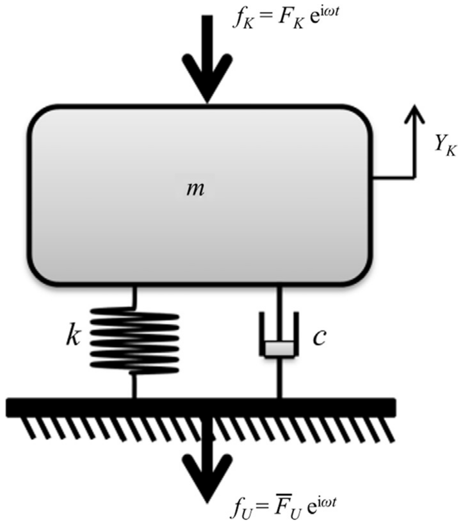

Fig. 3. Applied and transmitted forces in the sdof case.

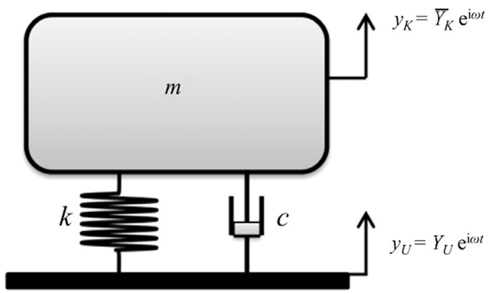

Fig. 4. The system of Fig. 3, here set free in space.

To verify Eq. (14), one needs to define the FRFs H_{UU} and H_{KU}. H_{UU} is equal to Y_U when a harmonic force of unitary magnitude is applied at coordinate U and no force is applied at coordinate K. Therefore, from Eq. (21),

$$\begin{Bmatrix} \overline{Y}_K \\ Y_U \end{Bmatrix} = \begin{bmatrix} k-\omega^2 m+\mathrm{i}\omega c & -k-\mathrm{i}\omega c \\ -k-\mathrm{i}\omega c & k+\mathrm{i}\omega c \end{bmatrix}^{-1} \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \tag{23}$$

Thus,

$$H_{UU} = Y_U = \frac{k-\omega^2 m+\mathrm{i}\omega c}{-\omega^2 m(k+\mathrm{i}\omega c)} \tag{24}$$

Likewise, H_{KU} is equal to $\overline{Y}_K$ when a harmonic force of unitary magnitude is applied at coordinate U and no force is applied at coordinate K. Therefore, Eq. (23) leads to

$$H_{KU} = \overline{Y}_K = \frac{1}{-\omega^2 m} \tag{25}$$

Applying Eq. (14) leads to

$$T^{(d)}_{UK} = H_{UU}(H_{KU})^{+} = \frac{Y_U}{\overline{Y}_K} = \frac{k-\omega^2 m+\mathrm{i}\omega c}{(k+\mathrm{i}\omega c)} \tag{26}$$

Using Eq. (17), it follows that

$$T^{(f)}_{UK} = -\frac{k+\mathrm{i}\omega c}{k-\omega^2 m+\mathrm{i}\omega c} \tag{27}$$

This expression is identical to Eq. (19) except for the minus sign; this is due to the fact that in literature the force transmissibility for the sdof system is normally calculated considering the transmitted force (as shown in Fig. 3) and Eq. (19). However, this study uses the opposite force, i.e., the reaction force, as considered in this paper (see Fig. 1). In any case, the result in terms of modulus amplitudes is obviously the same.

One should also note that, for the sdof case, what is usually defined as displacement transmissibility is the ratio between the amplitude of the response ($\overline{Y}_K$) to the imposed displacement amplitude at the base, (Y_U), i.e., the inverse of Eq. (26).

These considerations explain why, in the case of an sdof system, both displacement and force transmissibilities coincide.

3.2. Limitations of the transmissibility relations

As mentioned in the previous section, the existence of these relations and possible direct application are limited by the dimension of the sets of coordinates U and K. Let us analyze these limitations in each case, assuming that the transmissibility matrices are experimentally measured.

Case 1: $\#K = \#U$
It is the ideal case; it is possible to measure both transmissibilities experimentally and it is possible to apply relations (17) and (18) directly.
Case 2: $\#K > \#U$
It is possible to measure the force transmissibility, and one can directly apply relation (18) to obtain the displacement transmissibility from the force transmissibility. It is also possible to measure the displacement transmissibility; however, the opposite relation (17) cannot be applied using this matrix because the inverse of the displacement transmissibility matrix cannot be calculated when $\#K \geq \#U$. Anyway, it is still possible to measure the inverse of the displacement transmissibility experimentally and thus estimate the force transmissibility by Eq. (17). The procedure to measure the displacement transmissibility and its inverse will be explained in Section 5.3 of this study and also can be found in Reference [2].
Case 3: $\#K < \#U$
It is not possible to apply Eq. (18) directly due to the inverse of the force transmissibility matrix; in this case one needs to use more coordinates for the applied forces K, which can be accomplished by introducing the concept of fictitious forces, as will be explained in Section 3.2.1.

Eq. (17) shows that it is possible to obtain the force transmissibility from the displacement transmissibility, but one may find some difficulties in obtaining the displacement transmissibility matrix, because it does not fulfill the criterion imposed by the displacement transmissibility $\#K \geq \#U$ (Eq. (14)). A possible solution consists of adding fictitious coordinates K to obtain $\#K = \#U$. That will not change the transmissibility values, because they represent null forces on the force transmissibility, as will be explained in Section 3.2.1. If there is no interest to know the displacement transmissibility it is also possible to measure its inverse and use it directly in relation (17).

Please cite this article as: Y.E. Lage, et al., Force transmissibility versus displacement transmissibility, *Journal of Sound and Vibration* (2014), http://dx.doi.org/10.1016/j.jsv.2014.05.038

3.2.1. Fictitious coordinates and fictitious forces

One important point mentioned above that can help the implementation of the transmissibility relationships is the addition of coordinates K to the system. This will not change the results obtained by the transmissibilities, since one must not forget that the solution of our problem is just a sub-matrix of the resulting matrix. This can facilitate the measurement of the displacement transmissibility, fulfilling the criterion required for this. The added coordinates, described as “fictitious”, are associated to the dynamic responses at the set K of displacement transmissibility and to the forces at the set K of the force transmissibility.

Let us assume the following example: a dynamic system with three support points (null displacements), which results in three reactions, and with an external applied force, as illustrated in Fig. 5.

The force transmissibility and the associated coordinates are

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{bmatrix} T_{1,4}^{(f)} \\ T_{2,4}^{(f)} \\ T_{3,4}^{(f)} \end{bmatrix} \{F_4\} \text{ with } \mathbf{F}_U = \{F_1, F_2, F_3\} \text{ and } \mathbf{F}_K = \{F_4\} \tag{28}$$

To establish the relation between transmissibilities, the displacement transmissibility assumes the configuration shown in Fig. 6 and is given by

$$\begin{Bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{Bmatrix} = \begin{bmatrix} T_{1,4}^{(d)} \\ T_{2,4}^{(d)} \\ T_{3,4}^{(d)} \end{bmatrix} \{Y_4\} \text{ with } \mathbf{F}_A = \mathbf{F}_U = \{F_1, F_2, F_3\}, \ \mathbf{Y}_U = \{Y_1, Y_2, Y_3\} \text{ and } \mathbf{Y}_K = \{Y_4\} \tag{29}$$

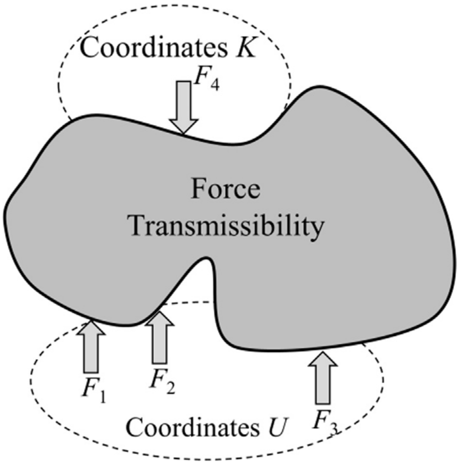

Fig. 5. Force transmissibility configuration.

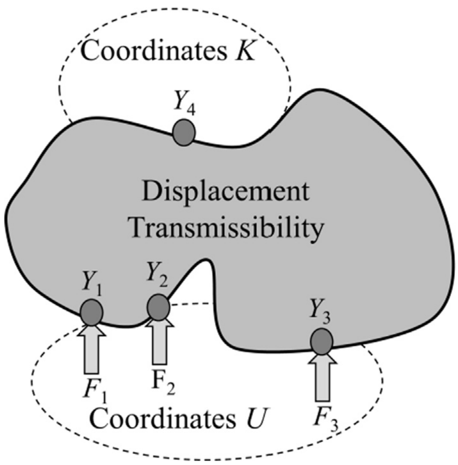

Fig. 6. Displacement transmissibility configuration.

As in the case of Fig. 6, one has $\#K < \#U$, and the displacement transmissibility cannot be calculated from Eq. (18); the idea is to add coordinates to the set K, such that $\#K = \#U$. Adding coordinates 5 and 6 to this example, as illustrated in Fig. 7, the displacement transmissibility is now given by

$$\begin{Bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{Bmatrix} = \begin{bmatrix} T_{1,4}^{(d)} & T_{1,5}^{(d)} & T_{1,6}^{(d)} \\ T_{2,4}^{(d)} & T_{2,5}^{(d)} & T_{2,6}^{(d)} \\ T_{3,4}^{(d)} & T_{3,5}^{(d)} & T_{3,6}^{(d)} \end{bmatrix} \begin{Bmatrix} Y_4 \\ Y_5 \\ Y_6 \end{Bmatrix} \tag{30}$$

Now, from Eq. (17) one obtains the force transmissibility with two additional forces, K, which are designated as fictitious, because they are null. The force transmissibility is defined as

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = ((\mathbf{T}_{UK}^{(d)})^+)^{\mathrm{T}} \begin{Bmatrix} F_4 \\ F_5 = 0 \\ F_6 = 0 \end{Bmatrix} \tag{31}$$

The coordinates corresponding to the fictitious forces are required to calculate the displacement transmissibility matrix, as will be seen in numerical and experimental examples. If there are no advantages in knowing the displacement transmissibility, one can directly obtain its inverse (inverse of Eq. (14)), and the addition of K coordinates is not necessary. Using the inverse matrix of the displacement transmissibility in Eq. (31) it follows that

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \left(\begin{bmatrix} H_{4,1} & H_{4,2} & H_{4,3} \\ (H_{5,1}) & (H_{5,2}) & (H_{5,3}) \\ (H_{6,1}) & (H_{6,2}) & (H_{6,3}) \end{bmatrix} \begin{bmatrix} H_{1,1} & H_{1,2} & H_{1,3} \\ H_{2,1} & H_{2,2} & H_{2,3} \\ H_{3,1} & H_{3,2} & H_{3,3} \end{bmatrix}^{-1} \right)^{\mathrm{T}} \begin{Bmatrix} F_4 \\ 0 \\ 0 \end{Bmatrix} \tag{32}$$

where the "( )" in the matrix entries means that those entries are not necessary, i.e.,

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{bmatrix} H_{1,1} & H_{1,2} & H_{1,3} \\ H_{2,1} & H_{2,2} & H_{2,3} \\ H_{3,1} & H_{3,2} & H_{3,3} \end{bmatrix}^{-1} \begin{bmatrix} H_{4,1} \\ H_{4,2} \\ H_{4,3} \end{bmatrix} F_4 \tag{33}$$

The multiplication of the receptance matrix in (32) reveals that the addition of coordinates K for the displacement transmissibility matrix does not change the sub-matrix relating the forces (when applying Eq. (17)), because each column of the final matrix only depends on a single coordinate K. A similar conclusion can be drawn for additional fictitious forces – each column of the force transmissibility matrix is dependent on only one force. In practice, the force transmissibility can be measured column by column, applying the forces of set K independently.

Summarizing, using the deductions presented before, and fulfilling the necessary requirements, one can use Eqs. (17) and (18) as follows:

i) when the displacements at the coordinates of the reactions are equal to zero ($\mathbf{Y}_U = \mathbf{0}$), one can write:

$$\mathbf{F}_U = -((\mathbf{T}_{UK}^{(d)})^+)^{\mathrm{T}} \mathbf{F}_K \tag{34}$$

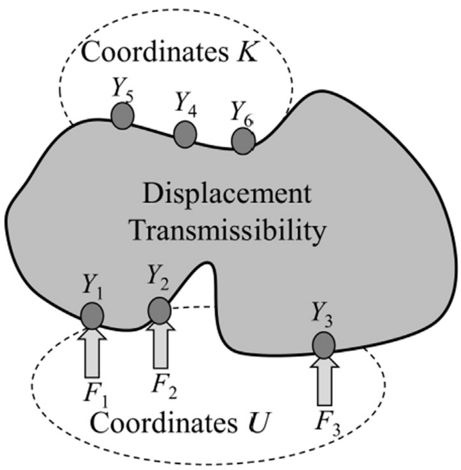

Fig. 7. Displacement transmissibility configuration with additional coordinates K.

and

$$\mathbf{Y}_U = -((\mathbf{T}_{UK}^{(f)})^{\mathrm{T}})^{+}\mathbf{Y}_K \tag{35}$$

ii) when the displacements at the coordinates of the reactions are not zero ($\mathbf{Y}_U \neq \mathbf{0}$), one can write:

$$\mathbf{F}_U = -((\mathbf{T}_{UK}^{(d)})^{+})^{\mathrm{T}}\mathbf{F}_K + (\mathbf{H}_{UU})^{-1}\mathbf{Y}_U \tag{36}$$

4. Numerical simulations

In this section, the authors present the methodologies used to test the proposed transmissibility relationships from the numerical point of view. These methodologies are here applied to simple beams.

4.1. Numerical models

To obtain the transmissibility needed in numerical simulation, one may consider the simulated FRF constituting the receptance matrix **H**. The receptance frequency response matrix **H** relates the dynamic displacement amplitudes **Y** with the external force amplitudes **F** as (using harmonic excitation, in steady-state conditions):

$$\mathbf{Y} = \mathbf{H}\mathbf{F} \Leftrightarrow \mathbf{Y} = (\mathbf{K} - \omega^2\mathbf{M} + \mathrm{i}\omega\mathbf{C})^{-1}\mathbf{F} \tag{37}$$

where **K**, **M** and **C** represent the stiffness, mass and proportional damping matrices, respectively, where $\mathbf{C}=\alpha\ \mathbf{K}+\beta\ \mathbf{M}$, and α and β are constants evaluated experimentally. **H** includes all the coordinates (degrees of freedom of the numerical model) in which the system is discretized and corresponds to the inverse of the dynamic stiffness matrix **Z**. Using (37), it is possible to obtain the FRFs; however, due to the inverse calculation being done for each frequency, the computational cost is fairly high. Instead, one shall use the FRFs written in terms of the modal properties. Taking into account the orthogonality properties, it follows that

$$\begin{gathered}\mathbf{\Phi}^{\mathrm{T}}\mathbf{M}\mathbf{\Phi} = \mathbf{I}\\ \mathbf{\Phi}^{\mathrm{T}}\mathbf{K}\mathbf{\Phi} = \mathbf{diag}(\omega_r^2)\\ \mathbf{\Phi}^{\mathrm{T}}\mathbf{C}\mathbf{\Phi} = \mathbf{diag}(2\xi_r\omega_r)\end{gathered} \tag{38}$$

where ω_r is the rth natural frequency, ξ_r is the rth damping factor and $\mathbf{\Phi}$ is the mass-normalized mode shape matrix. Then one obtains:

$$\mathbf{Y} = \mathbf{H}\mathbf{F} \Leftrightarrow \mathbf{Y} = \mathbf{\Phi}(\mathbf{diag}(\omega_{\mathbf{r}}^2 - \omega^2 + \mathrm{i}2\xi_{\mathbf{r}}\omega_{\mathbf{r}}\omega))^{-1}\mathbf{\Phi}^{\mathrm{T}}\mathbf{F} \tag{39}$$

In this way, one only needs to compute the inverse of a diagonal matrix. A computer program was developed (in MatLab® environment) to build the stiffness, mass and damping matrices, using the finite element method, and to perform the needed FRFs using (39).

4.2. Transmissibilities in terms of numerical simulations

As explained in the previous subsection, although the receptance matrix **H** is the inverse of the corresponding dynamic stiffness matrix, one should avoid such frequency by frequency direct numerical inversion. Instead, **H** is calculated as in Eq. (39), to avoid a high computational cost.

To verify Eq. (17), a finite element model has been chosen according to the theory considered for the structure discretization. For example, in the case of reasonably long and slender beams, one can use the standard two-node Bernoulli-Euler bidimensional beam element. With it, and as the analysis and the model are limited to the plane xOy (see Fig. 8), each node has three degrees of freedom (u_x, u_y, θ). Hence, the matrices of the numerical model have an order of $3 \times N$ for the free–free beam, where N is the total number of nodes.

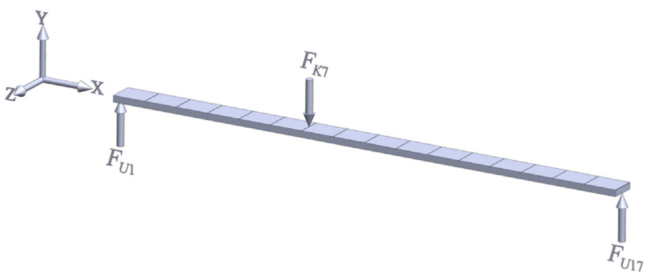

Fig. 8. Beam used for the force transmissibility example, with a transversal force at node 7 and reactions at nodes 1 and 17.

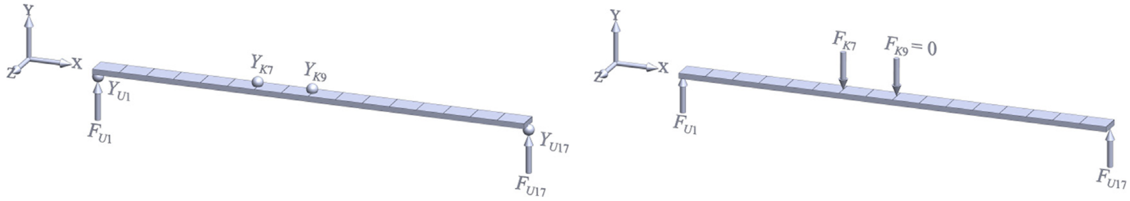

Fig. 9. Structural models after imposing the criterion $\#K=\#U$.

As far as the simulations are concerned, only the displacements and applied forces along the Oy direction are used and therefore the numbering of nodes coincides with the number of coordinates y.

To confirm numerically the relationships obtained in Eqs. (17) and (18), the following model is proposed: a beam simply supported at both ends is subjected to a transverse force somewhere along the beam axis. For the sake of simplicity, one considers the beam discretized in sixteen finite elements with supports at nodes 1 and 17, and a transverse dynamic force applied at node 7, as illustrated in Fig. 8.

The sets involved are $\mathbf{F}_U=\{F_1 \quad F_{17}\}$ and $\mathbf{F}_K=\{F_7\}$, related through the force transmissibility:

$$\begin{Bmatrix} F_1 \\ F_{17} \end{Bmatrix} = -\begin{bmatrix} H_{1,1} & H_{1,17} \\ H_{17,1} & H_{17,17} \end{bmatrix}^{-1} \begin{bmatrix} H_{1,7} \\ H_{17,7} \end{bmatrix} \{F_7\} \Leftrightarrow \begin{Bmatrix} F_1 \\ F_{17} \end{Bmatrix} = \begin{bmatrix} T^{(f)}_{1,7} \\ T^{(f)}_{17,7} \end{bmatrix} \{F_7\}. \tag{40}$$

The force transmissibility matrix can be determined using the receptance matrix or by the ratio between the simulated reactions and the applied forces. Now, the objective is to obtain the same relation between the forces but using the displacement transmissibility concept.

According to the description given in Section 3, it is not possible to relate the force transmissibility directly to the displacement transmissibility, as in Eq. (17) using the setup presented in Fig. 8, because $\#K<\#U$, and therefore the displacement transmissibility cannot be calculated. As explained before, if one does not need to know the displacement transmissibility matrix, it is possible to apply Eq. (17) using the inverse of the displacement transmissibility matrix directly. Assuming, in this case, that one would like to know the displacement transmissibility, one may add new coordinates to ensure that $\#K=\#U$, i.e., to add a fictitious coordinate to the displacement transmissibility matrix, which corresponds to a fictitious force in the force transmissibility matrix.

For example, let us chose coordinate 9 as the fictitious one. Considering the models illustrated in Fig. 9, the following sets of coordinates will be required for the displacement and force transmissibility, respectively:

$$\mathbf{T}^{(d)} \to \mathbf{Y}_U=\{Y_1 \quad Y_{17}\}, \mathbf{Y}_K=\{Y_7 \quad Y_9\} \text{ and } \mathbf{F}_A=\mathbf{F}_U=\{F_1 \quad F_{17}\} \tag{41}$$

$$\mathbf{T}^{(f)} \to \mathbf{F}_U=\{F_1 \quad F_{17}\} \text{ and } \mathbf{F}_K=\{F_7 \quad F_9\} \text{ with } F_9=0 \tag{42}$$

The transmissibility matrices are then defined by

$$\begin{Bmatrix} F_1 \\ F_{17} \end{Bmatrix} = -\begin{bmatrix} H_{1,1} & H_{1,17} \\ H_{17,1} & H_{17,17} \end{bmatrix}^{-1} \begin{bmatrix} H_{1,7} & H_{1,9} \\ H_{17,7} & H_{17,9} \end{bmatrix} \begin{Bmatrix} F_7 \\ F_9 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} F_1 \\ F_{17} \end{Bmatrix} = \begin{bmatrix} T^{(f)}_{1,7} & T^{(f)}_{1,9} \\ T^{(f)}_{17,7} & T^{(f)}_{17,9} \end{bmatrix} \begin{Bmatrix} F_7 \\ F_9 \end{Bmatrix} \tag{43}$$

$$\begin{Bmatrix} Y_1 \\ Y_{17} \end{Bmatrix} = \begin{bmatrix} H_{1,1} & H_{1,17} \\ H_{17,1} & H_{17,17} \end{bmatrix} \begin{bmatrix} H_{7,1} & H_{7,17} \\ H_{9,1} & H_{9,17} \end{bmatrix}^{-1} \begin{Bmatrix} Y_7 \\ Y_9 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} Y_1 \\ Y_{17} \end{Bmatrix} = \begin{bmatrix} T^{(d)}_{1,7} & T^{(d)}_{1,9} \\ T^{(d)}_{17,7} & T^{(d)}_{17,9} \end{bmatrix} \begin{Bmatrix} Y_7 \\ Y_9 \end{Bmatrix} \tag{44}$$

With this, it is possible to obtain both transmissibilities, as well as the relationships between them. The purpose of this is to illustrate how to calculate the transmissibilities using the numerical model (receptances) for a free–free beam, and numerically confirm relationships (17) and (18).

Here we have presented the relations between transmissibilities using the receptance, which may not be the most interesting example, as any of the transmissibilities can be calculated directly. The methodology using only the dynamic responses of the structure is more interesting and focuses on the main purpose of this article, i.e., the possibility of knowing a relationship between forces only from the measurement of displacements, as will be explained in Section 5.3.

4.3. Numerical results

The properties of the beam are presented in Table 1.

In order to confirm relationship (17) numerically, the several entries of the force transmissibility matrix are obtained directly from the numerical FRFs of the beam (Eq. (43)) and then compared with the force transmissibility matrix obtained through the application of Eq. (17) to the displacement transmissibility matrix (here also obtained directly from the FRFs). As can be seen from Fig. 10, both curves are coincident.

Table 1
Beam properties.

Young's modulus – E	208 GPa
Density – ρ	7840 kg/m^3
Length – L	0.8 m
Section width – b	5.0×10^{-3} m
Section height – h	20.0×10^{-3} m
Second moment of area – I_{zz}	2.0833×10^{-10} m^4

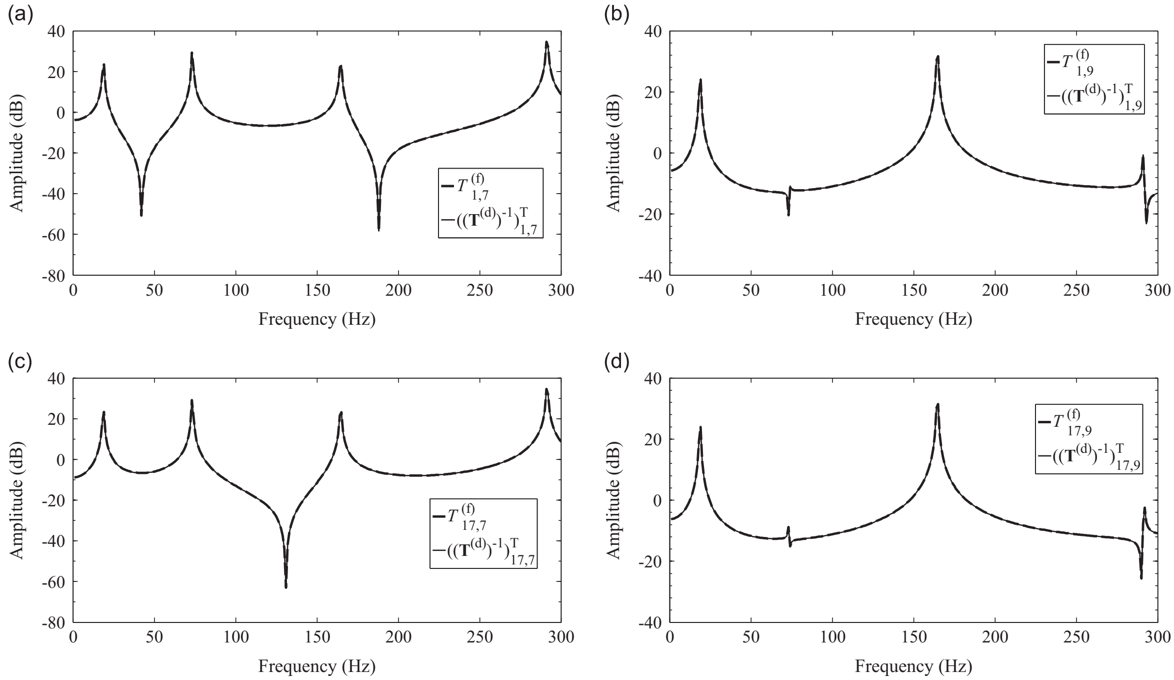

Fig. 10. Comparison of results obtained numerically for the transmissibility by (21,37,38): (a) $T_{1,7}$; (b) $T_{1,9}$; (c) $T_{17,7}$; and (d) $T_{17,9}$.

5. Experimental tests

In this section, the authors present the methodologies used to test the proposed transmissibility relationships from the experimental point of view, applied once again to simple beams.

5.1. Transmissibilities in terms of experimental measurements

Concerning the force transmissibility and assuming a general case with multiple applied forces and multiple reactions, the experimental measurement of the elements of its matrix is considered simple. In this case, one can measure the elements of the force transmissibility matrix, column by column, individually applying each input corresponding to the respective column. This is possible since each column of the force transmissibility depends only on a single input (Eq. (4)).

Direct measurement of the displacement transmissibility matrix is not straightforward. A detailed description of the indirect procedure applied in this study is given in Section 5.3.

For the experimental tests, a steel beam with rectangular cross-section (the properties presented in Table 1) was chosen. All measurements were conducted only in the y-direction; the structure is in a free–free configuration for the displacement transmissibility study and is simply supported at two points for the force transmissibility study. Once again, the main objective is to test and validate relationships (17) and (18).

5.2. Force transmissibility measurement

Fig. 11 presents a schematic representation of the experimental setup used for the force transmissibility tests. The simply supported beam has a single applied force, so the chosen sets of forces are $\mathbf{F}_U = \{F_1 \quad F_{17}\}$ and $\mathbf{F}_K = \{F_7\}$. The supports of the

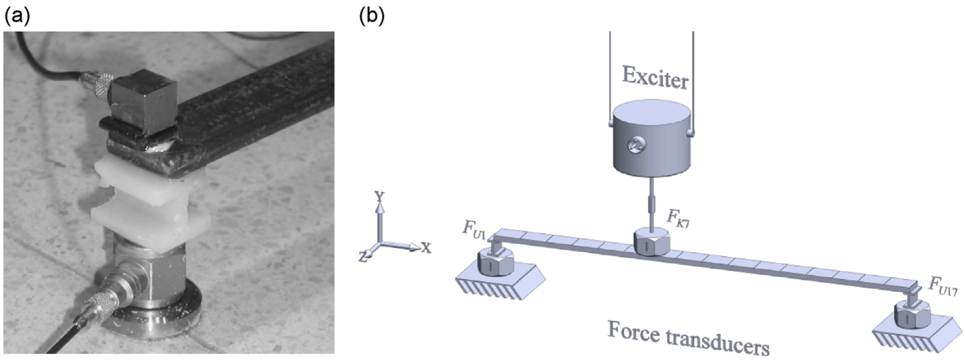

Fig. 11. (a) Nylon support used to support the beam at its ends and (b) experimental setup for the transmissibility of forces with positions of the force transducers.

beam are simulated by small, wide-flanged shapes (built of nylon) shown in Fig. 11, preventing vertical displacements of the ends while allowing free rotation along the Oz axis. The excitation signal used was a multi-sine transmitted to the exciter, with constant amplitude in frequency. In reality, the signals measured by the force transducer vary in amplitude with the frequency, as it also depends on the dynamic response of the structure.

The transmissibility matrix relates the transverse applied force with the reactions:

$$\begin{Bmatrix} F_1 \\ F_{17} \end{Bmatrix} = \begin{bmatrix} T^{(f)}_{1,7} \\ T^{(f)}_{17,7} \end{bmatrix} \{F_7\} \tag{45}$$

5.3. Displacement transmissibility measurement

To determine the displacement transmissibility and apply Eq. (17) one needs to meet the criterion $\#K = \#U$. For that, it is necessary to add a fictitious coordinate to the set of coordinate's responses, as explained before. Similarly to the numerical case, the setup is as illustrated in Fig. 12 with the following sets of coordinates: $\mathbf{Y}_U = \{Y_1 \quad Y_{17}\}$, $\mathbf{Y}_K = \{Y_7 \quad Y_9\}$ and $\mathbf{F}_A = \mathbf{F}_U = \{F_1 \quad F_{17}\}$.

$$\begin{Bmatrix} Y_1 \\ Y_{17} \end{Bmatrix} = \begin{bmatrix} T^{(d)}_{1,7} & T^{(d)}_{1,9} \\ T^{(d)}_{17,7} & T^{(d)}_{17,9} \end{bmatrix} \begin{Bmatrix} Y_7 \\ Y_9 \end{Bmatrix} \tag{46}$$

Thus, the results obtained should enable one to test the proposed relationship (17):

$$\mathbf{T}^{(f)} = -((T^{(d)})^+)^{\mathrm{T}} \Leftrightarrow \begin{Bmatrix} F_1 \\ F_{17} \end{Bmatrix} = ((\mathbf{T}^{(d)}_{UK})^+)^{\mathrm{T}} \begin{Bmatrix} F_7 \\ F_9 = 0 \end{Bmatrix} \tag{47}$$

where the force transmissibility corresponds to the first column of the matrix obtained by this relationship while the other column corresponds to the related fictitious force at the added coordinate.

To measure the respective displacement transmissibility matrix and to avoid measuring the forces (i.e., no force transducers) one can use several operating measurements, like the one proposed in [2,30].

Note that the displacement transmissibility matrix can be obtained by exciting the structure for each force input individually. For each applied force of the vector $\mathbf{F}_U$, one obtains the responses at subset $U[\{Y_1{}^{(1)}...Y_U{}^{(1)}\}^{\mathrm{T}}...\{Y_1{}^{(U)}...Y_U{}^{(U)}\}^{\mathrm{T}}]$ and they are related to responses measured at subset $K\,[\{Y_1{}^{(1)}...Y_K{}^{(1)}\}^{\mathrm{T}}...\{Y_1{}^{(U)}...Y_K{}^{(U)}\}^{\mathrm{T}}]$ via the transmissibility matrix (which does not depend on loading but depends on the structure of the problem). These measurements can be combined into one matrix:

$$[\{Y_U\}^{(1)}\ldots\{Y_U\}^{(U)}] = [T^{(d)}_{UK}][\{Y_K\}^{(1)}\ldots\{Y_K\}^{(U)}]. \tag{48}$$

and $\mathbf{T}^{(d)}$ can be obtained as

$$[T^{(d)}_{UK}] = [\{Y_U\}^{(1)}\ldots\{Y_U\}^{(U)}][\{Y_K\}^{(1)}\ldots\{Y_K\}^{(U)}]^{-1} \tag{49}$$

The transmissibility matrix can then be obtained from this result and the settings necessary to test it in the laboratory are the ones illustrated in Fig. 13.

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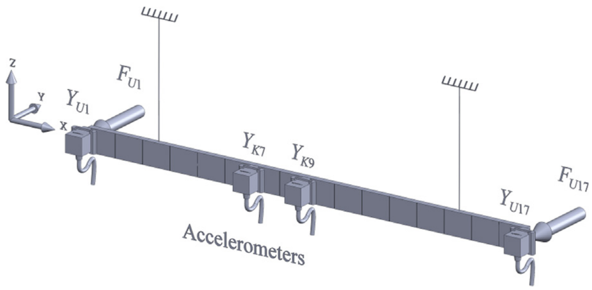

Fig. 12. Accelerometers and force positions for the displacement transmissibility with the additional coordinate Y_9 (suspended by nylon wires).

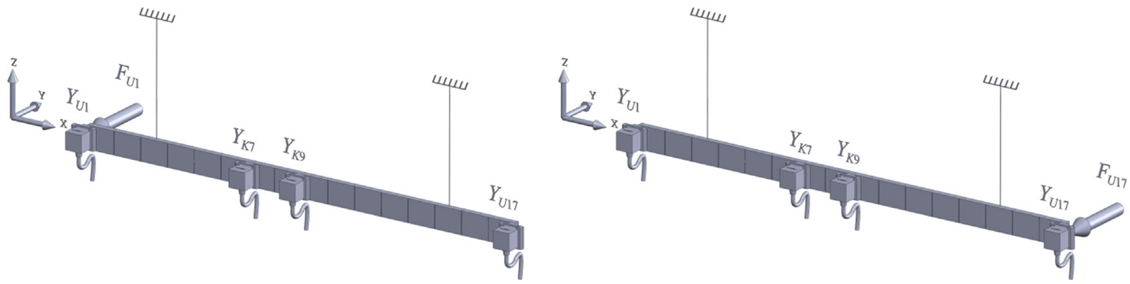

Fig. 13. Setups used for calculation of $\mathbf{T}^{(d)}$: on the left scheme the force transducer is at coordinate 1, while on the right scheme the transducer is at coordinate 17.

With the setups illustrated in Fig. 13 one can obtain the displacement transmissibility matrix as follows:

$$\mathbf{T}_{UK}^{(d)} = \begin{bmatrix} T_{1,7}^{(d)} & T_{1,9}^{(d)} \\ T_{17,7}^{(d)} & T_{17,9}^{(d)} \end{bmatrix} = \begin{bmatrix} Y_1^1 & Y_1^{17} \\ Y_{17}^1 & Y_{17}^{17} \end{bmatrix} \begin{bmatrix} Y_7^1 & Y_7^{17} \\ Y_9^1 & Y_9^{17} \end{bmatrix}^{-1} \tag{50}$$

This technique avoids the measurement of the amplitude of forces; it only requires the dynamic responses and uses of one vibration exciter.

Note that, as explained before, it is also possible to apply Eq. (17) without the knowledge of the displacement transmissibility matrix. In that case, one needs to obtain the inverse of the displacement transmissibility matrix (13) directly, needing no additional fictitious coordinates:

$$(\mathbf{T}_{UK}^{(d)})^{+} = \begin{bmatrix} Y_7^1 & Y_7^{17} \end{bmatrix} \begin{bmatrix} Y_1^1 & Y_1^{17} \\ Y_{17}^1 & Y_{17}^{17} \end{bmatrix}^{-1} \tag{51}$$

5.4. Experimental results

Here we present the displacement transmissibility matrix obtained using the setup shown in Fig. 13 and the components calculated using Eq. (50). The results are in the plots shown in Fig. 14a–d.

After obtaining the displacement transmissibility, one can obtain the required force transmissibility using Eq. (17), where the elements of the first column of the matrix correspond to the elements of the force transmissibility matrix measured experimentally. Of course, the force at coordinate 9 is a fictitious force, i.e., equal to zero. The plots in Fig. 15 show the two elements of the force transmissibility matrix and those obtained by the displacement transmissibility applying Eq. (17).

The curves obtained in both ways coincide reasonably well, although there are some discrepancies due to the experimental nature of the procedure used to obtain the displacement transmissibility. As can be seen in the displacement transmissibilities presented in Fig. 14, some small peaks appear, caused by the fact that in both measurement tests illustrated in Fig. 12 the mass of the force transducer has not been canceled out and therefore the resonant frequencies of the structure from those measurements were not exactly coincident.

Additional measurements also confirmed that the referred peaks of Fig. 14 were not due to effects of the degrees of freedom that have not been taken into account – namely transverse vibrations along another axis, torsional vibrations, etc.

From Fig. 15, one can also conclude that the adopted setup for the simple supported beam has proven to be acceptable, due to the good results that were obtained, although the nylon supports have introduced damping into the system. Moreover, the assumption of null displacements at the coordinates of the supports was also confirmed experimentally.

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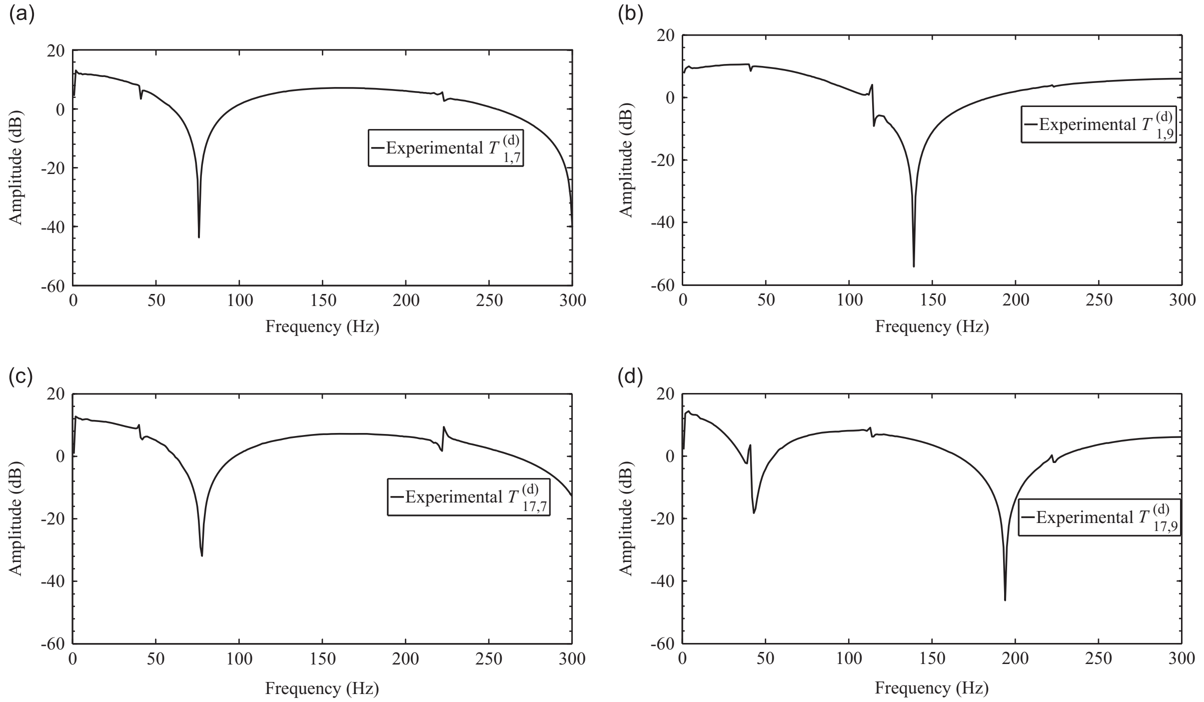

Fig. 14. Components of the displacement transmissibility matrix: (a) $T^{(d)}_{1,7}$, (b) $T^{(d)}_{1,9}$, (c) $T^{(d)}_{17,7}$, and (d) $T^{(d)}_{17,9}$ calculated from the experimental responses.

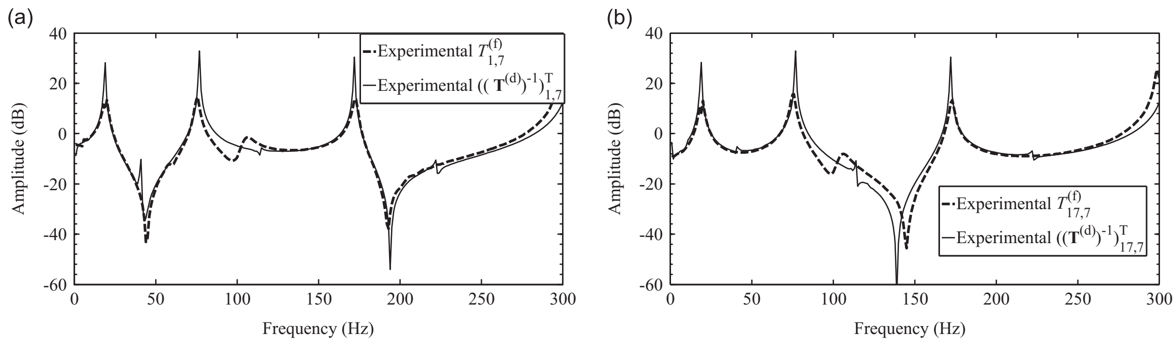

Fig. 15. (a) First element of force transmissibility $T^{(f)}_{1,7}$ and (b) second element of force transmissibility $T^{(f)}_{17,7}$.

With these examples, the authors consider that the main purpose of showing the effectiveness of the proposed relation has been achieved.

6. Conclusions

In this work, the authors derive and validate a relationship to obtain the force transmissibility from the displacement transmissibility and vice versa, in mdof dynamical systems. This relationship will allow one to perform force identification using displacement transmissibility, which is a very practical technique, especially if the displacement transmissibility can be measured in operational conditions, not using FRFs. In this case, it is not necessary to measure the forces and the study of the structure can be performed under free–free conditions.

The proposed relationship is validated through the numerical simulations and experimental tests. From these developments, one can draw the following main conclusions:

(i) In mdof systems, it is possible to establish a mathematical expression to obtain the displacement transmissibility from the force transmissibility and vice versa.

(ii) The existence of such an mdof transmissibility relation depends on the number of coordinates involved. In some situations the use of the inverse displacement transmissibility matrix, fictitious coordinates or fictitious forces can circumvent some restrictions in the application of Eqs. (17) and (18).
(iii) The present generalization naturally accommodates the well-known single-degree-of-freedom case.

By the proposed relationship the authors present a solution for an unanswered question in the field of the transmissibility concept.

Acknowledgments

The current investigation had the support of national funds through the Foundation for Science and Technology (FCT), under the Project PEst/OE/EME/LA0022/2011.

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