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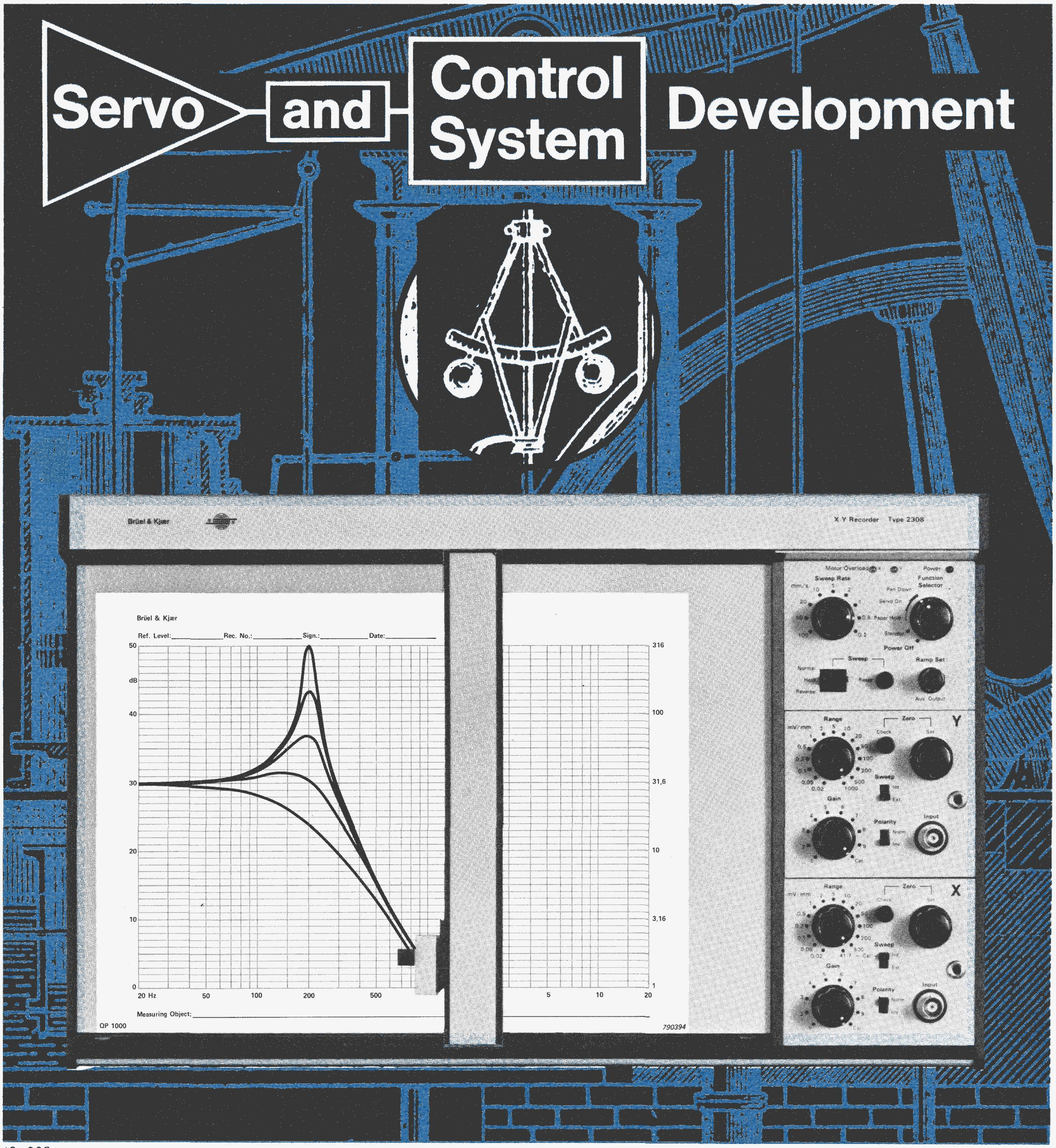
Practical Measurement Techniques for Servo and Control System Development



Practical Measurement Techniques for Servo and Control System Development

by *Philip Hollingbery*

Introduction

The applications of control theory are so extensive that the very language used for the same ideas differs from one field to the next. One man's control system is another man's servomechanism. There may seem little connection between the governor of a diesel engine, the ballcock of a water cistern and the negative feedback in a hi-fi amplifier — yet they are all common examples of closed-loop control systems.

The essence of a closed loop control system is shown in Fig.1.

The Output X_2 is some variable which we wish to control, the Demand X_1 is some convenient analogue of the output (such as the position of a knob which we can turn), and the loop in between is composed of components with deterministic relationships between their ports (i.e., inputs and outputs). The entire characteristics of the system are fixed by these components, and this note describes how the important properties can most easily be measured.

If all the components are linear — that is to say, the output is always proportional to the sum of the inputs — then the calculations and measurements are fairly easy but the system is impossible to construct in the real world. The components may, however, be linear over their working range, or have characteristics sufficiently well-behaved to permit linear theory to be applied (in which case the characteristics are known as small-signal characteristics). This is the most practical situation. Finally, the system may include completely non-continuous components — e.g., devices which switch on and off at a rate which is not fast compared with the response time of the system — and for such servos the methods described below are not so helpful.

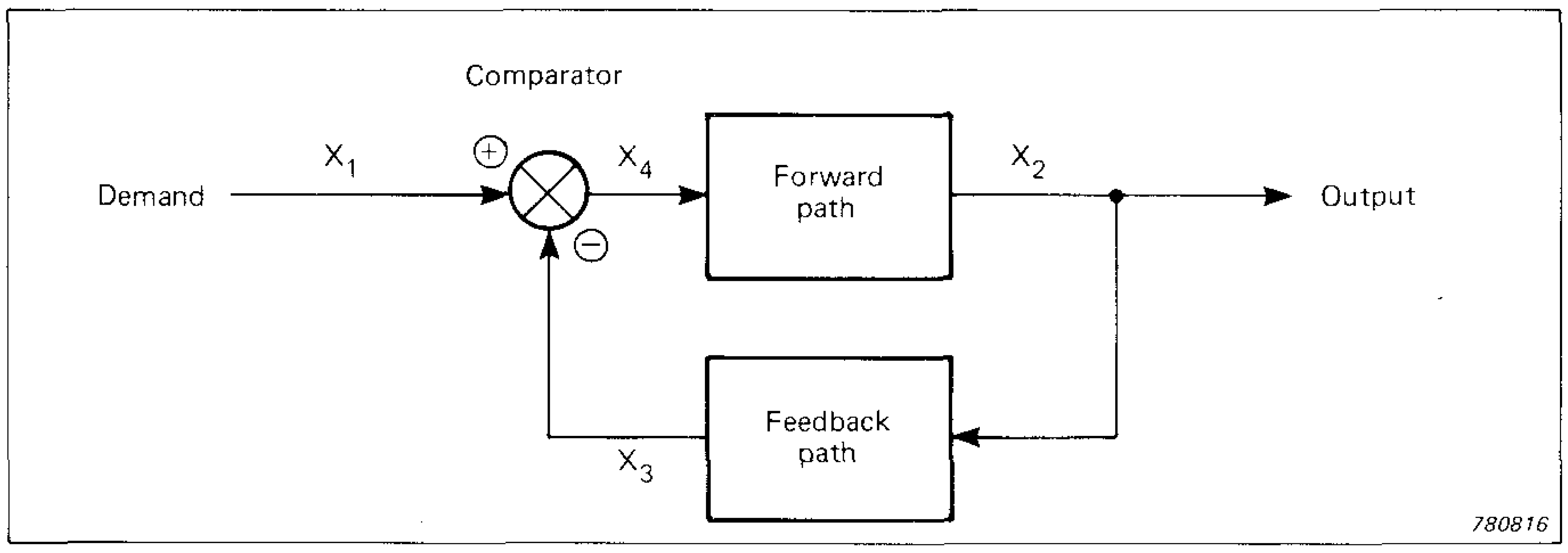


Fig.1. Basic closed-loop control system

Static Control Loop Analysis

Suppose the forward path in Fig.1 is such that the output is G times its input; and the output of the feedback path is H times **its** input.

Then $X_2 = GX_4$ (1)

and $X_3 = HX_2$; (2)

But $X_4 = X_1 - X_3$

$= X_1 - HX_2$. (3)

$X_2 = G(X_1 - HX_2)$, from (1);

$GX_1 = X_2 + GH_2$

$= X_2(1 + GH)$;

$$\frac{X_2}{X_1} = \frac{G}{1 + GH} \quad (4)$$

Equation 4 is known as the system equation.

The ratio of X_2/X_4 is known as the transfer coefficient of the forward path (and similarly for any other element which has an input and an output). If X_2 and X_4 are similar variables (e.g., both voltage), or there is known to be power amplification inside the element referred to, the transfer coefficient may also be described as the gain.

If $G \to 0$, then X_2/X_1 also $\to 0$, showing that the system becomes useless since the output does not respond to any demand.

If $H \to 0$, then $X_2/X_1 \to G$. This represents a simple open-loop control system.

If $GH >> 1$,

then $$\frac{X_2}{X_1} = \frac{1}{H} \text{ approx.} \quad (5)$$

This very useful result arises when the loop components are such that, if the loop is broken (i.e., open-loop conditions), there is overall gain large compared with 1. Since the open-loop gain is by definition the ratio of variables of the **same** kind (e.g., voltage, pressure) at the **same** impedance, overall gain greater *than 1 requires actual power gain* (as opposed to just voltage gain, for example).

The usefulness of equation (5) lies in the fact that components providing power amplification very often suffer from imperfections in their characteristics (noisiness, reduced frequency response, thermal variations, non-linearity, etc.), so if *we put all the loop gain into the forward path* we can construct the feedback path from passive components with nearly ideal characteristics - and provided $GH >> 1$ the transfer coefficient $X2/X1$ of the system becomes $1/H$, which is independent of G. Thus a closed-loop control system can be immunized against all kinds of perturbing influences *in a way which is not possible* for an open-loop system.

Dynamic Limitations of Static Analysis

However, all practical components include parasitic properties (such as finite mass in mechanical servos, and stray capacitance in electronic circuits) which make it ne*cessary to introduce time as a variable* in the system equation. In other words, no control system component is infinitely fast, so for a complete understanding it is necessary to analyze the dynamic characteristics of the system.

Even if the system designer knows that all the components he intends to use have dynamic responses which are very fast compared with the rapidity of response he requires, it is still almost always essential to determine the actual dynamic characteristics of each constituent component and then implement measures to ensure that the control system is sufficiently stable. Unstable control systems exhibit oscillation of large magnitude, or at best respond to transients by ringing, or overshooting. This behaviour usually prevents proper operation of the system, and is very likely to occur unless the designer takes systematic measures to prevent it.

These measures to ensure stability involve the deliberate inclusion of elements with controlled dynamic responses in the control loop. Examples of such elements are velocity feedback and integrating-networks. Such measures cannot be taken, however, without the requisite knowledge of the existing dynamic response characteristics.

The dynamic response is allowed for in the system equation by the introduction of terms containing the differential operator D (differentiation with respect to time). Integration is then represented by $1/D$. The system equation then becomes a linear differential equation which may be solved using Laplace transform methods.

Frequency Response Methods

In an ideal design situation, the designer has enough data on all the proposed components to write down the system equation as a *linear differential equation.* However, more often than not it will be necessary to measure the dynamic characteristics of some of the components. This involves applying an input signal, or excitation, to a component input, and measuring the response at the output. The most useful general-purpose excitation signal is a sinewave.

This is because of the unique features of sinusoidal excitation, i.e.,

1) the output (of a linear system) will be a sinewave of identical frequency;

2) the phase, amplitude and frequency of a sinewave can all be measured in an unambiguous, more or less straightforward, fashion since time can be eliminated from the measurement;

3) taking these measurements on a linear component enables its dynamic response to be completely defined;

4) the system equation can be rewritten in terms of steady-state sinusoidal response simply by replacing D with $j\omega$ (where j is $\sqrt{-1}$ and ω is the sinewave frequency multiplied by 2π).

Methods of measuring or expressing system characteristics as function of a sinusoidal frequency are known as frequency response methods.

Not only are frequency response methods the easiest methods for most practical measurement, they are also the most convenient in *many respects for the mathematical* analysis which accompanies practical measurement.

The system equation should therefore be rewritten:

$$\frac{X_2(\omega)}{X_1(\omega)} = \frac{G(\omega)}{1 + G(\omega)H(\omega)} \qquad (6)$$

It is relatively straightforward to predict the frequency response of a subsystem from its system equation, but it can be difficult to deduce the equation from a set of frequency response measurements, unless the number of terms in $j\omega$ is small. However, breaking the system down into its components for measurement purposes usually makes it easier to deduce the coefficients of the equation. This is also easier when experience has been gained of a range of responses of different system types.

$X_2(\omega)/X_1(\omega)$ is known as the Laplace transform, or sometimes as the transfer function, of the system. It is a complex number, consisting of real and imaginary parts. In many situations it is convenient to work with the modulus, or absolute value, of this quantity. When this modulus is plotted against ω (or against $\omega/2\pi$, the frequency) the resulting curve is known as the transfer characteristic, or the (amplitude) frequency response.

Since signal generators are usually graduated in Hertz rather than radians/s, it is common to work with the frequency rather than ω.

The Problem of Stability

Consider the loop of Fig.1 broken at any point, and a sinewave applied to the point immediately after the break. Provided the sinewave has a low frequency, and a moderately small amplitude (the latter requirement is important in slew-rate limited and nonlinear systems), the output at the point before the break will be a sinewave GH times the applied sinewave and of opposite phase (assuming there are no differentiators or integrators in the loop).

If the frequency of the sinewave is now increased, the relative phases of the input and output will begin to change, so that the two sinewaves will tend towards the same phase. This is an inevitable consequence of the use of physically realizeable (as opposed to idealized) components. It will also be observed that the sinewave at the output begins to diminish in amplitude as the frequency is increased.

A frequency will eventually be reached at which the output sinewave is less than 90° removed in phase from the input — that is, it may be resolved into two components of which one is in phase with the input sinewave. If this resolved component should also have a greater amplitude than that of the input sinewave, then closing the loop would result in behaviour of a kind not intended, since any energy present in the loop at this particular frequency (caused, for example, by a switch-on transient) would be progressively amplified as it passed around the loop, leading either to sustained oscillation or the destruction of the output components.

Even if this does not happen, lack of attention to the dynamic behaviour (i.e., frequency characteristics) of servo components can result in an inconvenient tendency to "ring" at a certain upper frequency.

The classic criterion for the stability of a closed-loop control system is Bode's Criterion, which may be summarized by saying that the open-loop gain of such a system must be less than 1,0 at the frequency for which the phase-shift round the loop becomes 180° — or, alternatively, that the phase-shift round the loop must be less than 180° at the frequency at which the open-loop gain falls to unity.

The value of the open-loop gain at the frequency for which the phase-shift becomes 180° is known as the gain margin. Similarly, the phase-shift (relative to 180°) at the frequency for which the open-loop gain becomes 0 dB (unity) is known as the phase margin. See Fig.2.

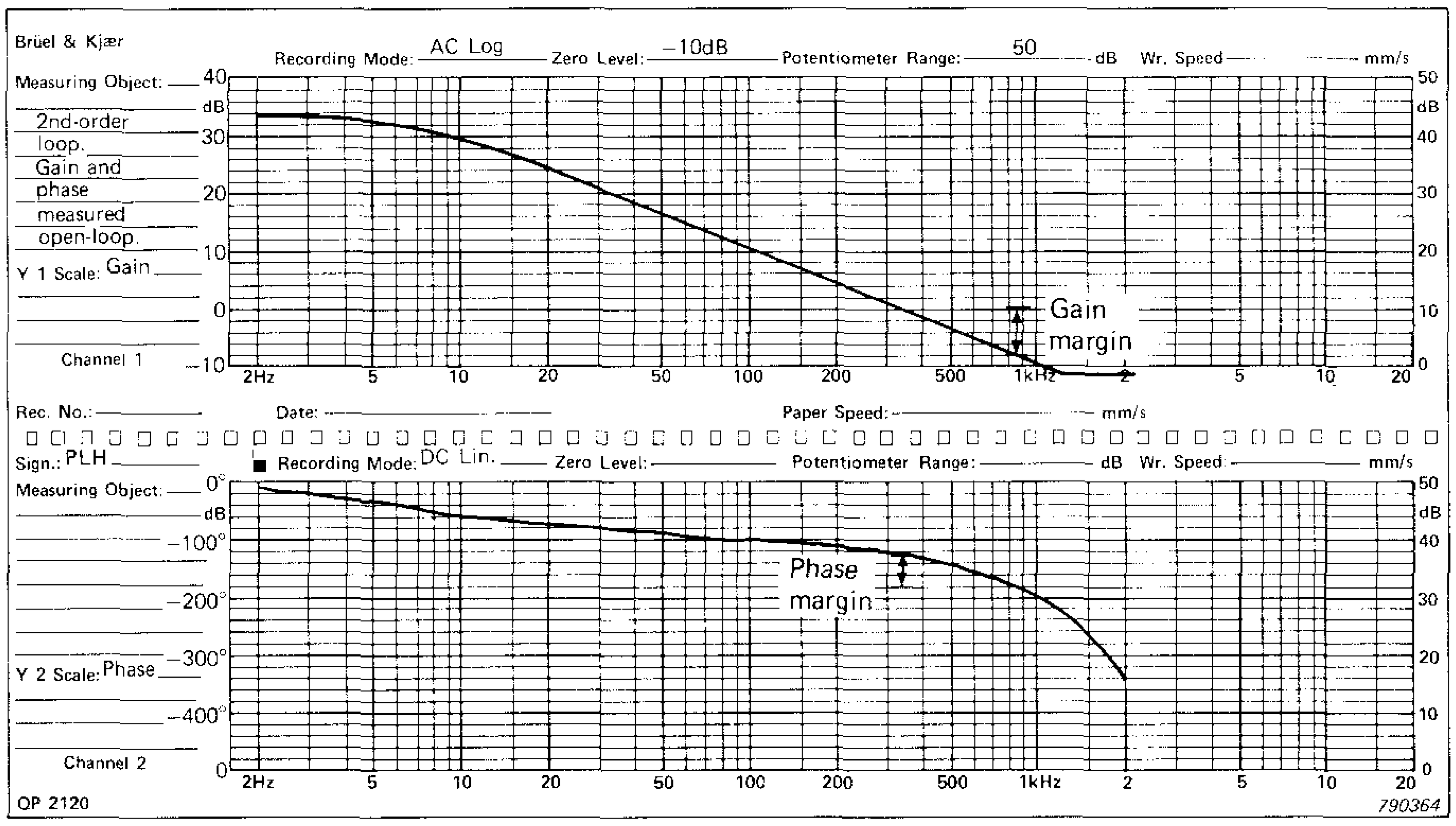


Fig.2. Recordings of open-loop gain and phase of a control loop, showing gain and phase margins. This kind of graph is sometimes described as a Bode plot

The Designer's Task

Many kinds of closed-loop control systems do not permit actual measurements to be made open-loop, because closed-loop operation is essential either to keep one or more of the components operating within its linear range, or to prevent the output devices from damaging or destroying themselves. Examples of such systems are the phase locked loops used in telecommunications, and power electronics servos used to control electrical machines.

On the other hand these are often the systems where the control engineer is required to incorporate system components which already exist and for which no one can supply reliable frequency-response data.

The task of the designer is therefore: (1) to ascertain, by measurement where necessary, the frequency and phase responses of those components on whose characteristics he can exercise little or no influence, and (2) to design the remainder of the loop in such a way as to satisfy both the performance requirements and the Bode Criterion.

There are other criteria and analytical methods for dealing with the problem of stability — such as Nyquist's Criterion, and the Root Locus Plot. However, most of them start off from the basis of sinusoidal steady state data, and that is what most of the methods discussed here are designed to measure.

Presentation of Frequency Response Data

Control loop data is often expressed graphically in the form of a Bode Plot, on which frequency is plotted logarithmically along the horizontal axis of a graph, and gain is plotted logarithmically against the vertical axis as a function of frequency. Phase may also be plotted on the same graph, although for many control systems (minimum phase systems) the phase response can be deduced readily from the gain plot, 90° phase lag being associated with a gradient of —1 and 180° with a gradient of —2, provided the same logarithmic scales are used on both axes.

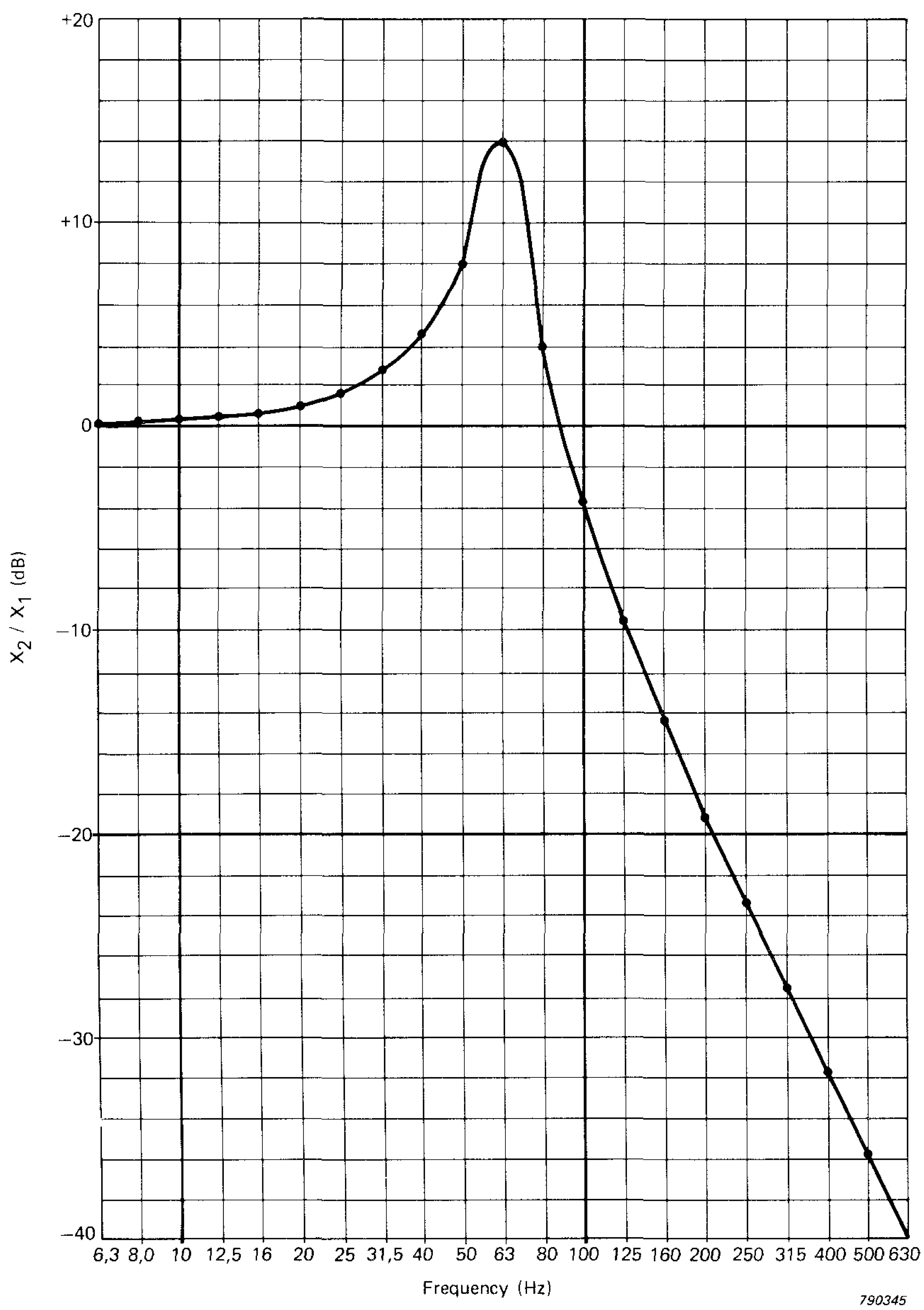


Fig.3. Example of a Bode plot of amplitude, made by hand

Gain is usually expressed in decibels (dB) for control system analysis. Thus for Fig.1

$$\text{Gain in dB} = 20 \log \frac{X_2}{X_1}.$$

The use of dB enables the overall gain of several cascaded components to be calculated simply by the **addition** of the constituent gains (provided these are expressed in

$\frac{X}{X_{REF}}$	dB
0,001	−60
0,01	−40
0,1	−20
1,0	0
1,1	1
1,3	2
1,4	3
1,6	4
1,8	5
2,0	6
2,2	7
2,5	8
2,8	9
3,2	10
3,5	11
4,0	12
4,5	13
5,0	14
5,6	15
6,3	16
7,1	17
7,9	18
8,9	19
10	20
100	40
1000	60
10000	80
100000	100
1000000	120

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Table 1. Conversion from amplitude ratios to decibels (dB) and vice versa

dB), and this may be done graphically for several frequency response plots. If gains were expressed in arithmetic units, they would have to be multiplied to arrive at the overall figure.

Another great advantage of plots which are logarithmic in both amplitude and frequency is that certain common features of linear systems can readily be recognized, because the basic system elements — differentiators and integrators — produce straight lines of unit slope on such plots.

Examples of Bode Plots are shown in Figs.2 and 3, and a ratio-to-dB conversion scale in Table 1.

Manual Frequency Response Measurement

The simplest way to measure the frequency response is to apply an actual sinewave to the input of the component whose characteristics are to be determined, and measure the amplitude of the resulting sinewave at the output at a number of different frequencies. Provided the input and output variables are both electrical in form, this may be done simply using the arrangement shown in Fig.4, which requires a sine generator and an AC voltmeter. It is helpful if the sine generator includes a built-in voltmeter to measure its own output, as it sometimes proves impracticable to use the same input voltage at every frequency to be measured. This is because at higher frequencies the exciting voltage has to be raised to provide an output voltage measurable above the noise.

It is also a great advantage if the AC voltmeter includes a dB scale, since this reduces the need to convert voltage ratios to dB.

The sine generator frequencies should correspond with the internationally preferred numbers listed in Table 2, selected according to the resolution required from the plot. The advantages of these numbers are that they lie at equal intervals along a logarithmic frequency scale and enable the Bode Plot to be recorded on ordinary squared paper, with the engineer's own choice of frequency range. This practice saves the need to stock a range of logarithmic graph papers.

The results of the measurement are set out in a table, prior to plotting. Table 3 is an example from which Fig.3 was obtained.

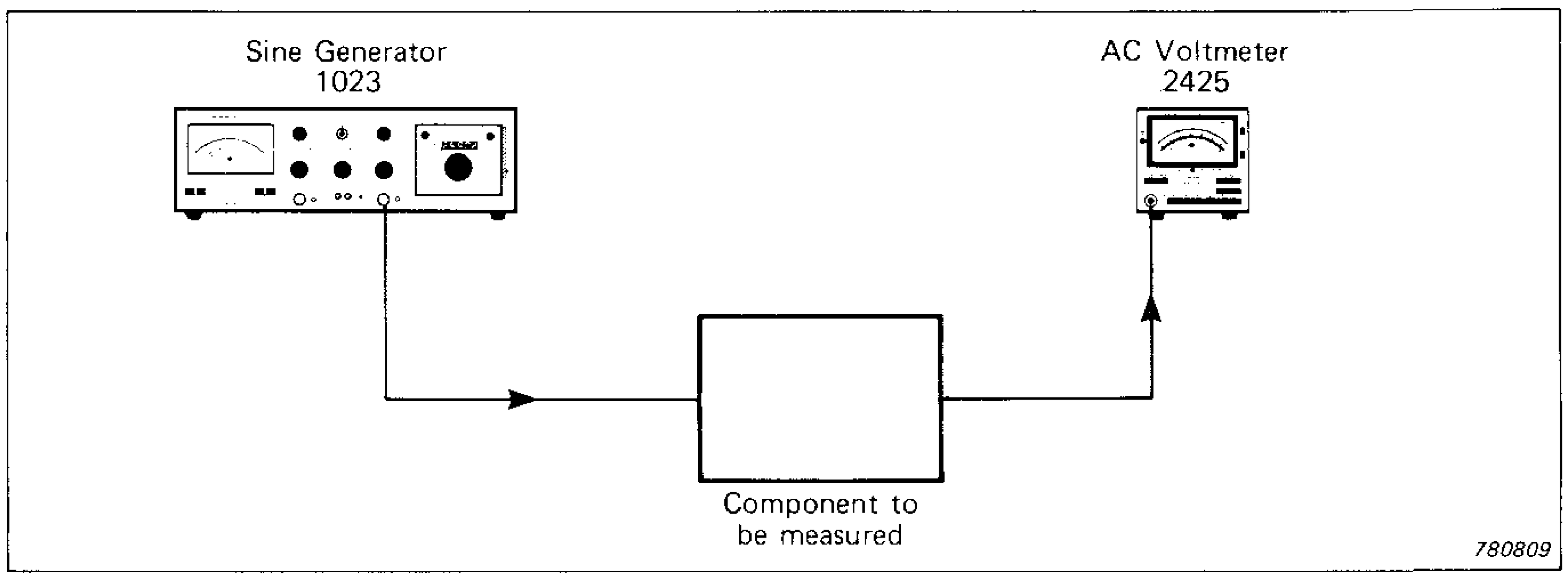

Fig.4. Simplest arrangement of equipment for measuring frequency response of servo component or subsystem

40-series	20-series	10-series	5-series	Exact Value	Mantissa
1	**1**	**1**	**1**	10000	000
1,06				10593	025
1,12	**1,12**			11220	050
1,18				11885	075
1,25	**1,25**	**1,25**		12589	100
1,32				13335	125
1,4	**1,4**			14125	150
1,5				14962	175
1,6	**1,6**	**1,6**	**1,6**	15849	200
1,7				16788	225
1,8	**1,8**			17783	250
1,9				18836	275
2	**2**	**2**		19953	300
2,12				21135	325
2,24	**2,24**			22387	350
2,36				23714	375
2,5	**2,5**	**2,5**	**2,5**	25119	400
2,65				26607	425
2,8	**2,8**			28184	450
3				29854	475
3,15	**3,15**	**3,15**		31623	500
3,35				33497	525
3,55	**3,55**			35481	550
3,75				37584	575
4	**4**	**4**	**4**	39811	600
4,25				42170	625
4,5	**4,5**			44668	650
4,75				47315	675
5	**5**	**5**		50119	700
5,3				53088	725
5,6	**5,6**			56234	750
6				59566	775
6,3	**6,3**	**6,3**	**6,3**	63096	800
6,7				66834	825
7,1	**7,1**			70795	850
7,5				74989	875
8	**8**	**8**		79433	900
8,5				84140	925
9	**9**			89125	950
9,5				94406	975

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Table 2. Internationally preferred numbers

Frequency (Hz)	Input Voltage X_1	Output Voltage X_2	X_2/X_1	dB
6,3	1	1,01	1,01	0,1
8,0	1	1,02	1,02	0,1
10	1	1,03	1,03	0,2
12,5	1	1,04	1,04	0,3
16	1	1,07	1,07	0,6
20	1	1,11	1,11	0,9
25	1	1,18	1,18	1,5
31,5	1	1,32	1,32	2,4
40	1	1,63	1,63	4,2
50	1	2,49	2,49	7,9
63	1	5,00	5,00	14,0
80	1	1,57	1,57	3,9
100	3,16	2,05	0,65	−3,8
125	3,16	1,05	0,33	−9,6
160	10	1,90	0,19	−14,5
200	10	1,11	0,111	−19,1
250	10	0,67	0,067	−23,5
315	10	0,41	0,041	−27,7
400	10	0,26	0,026	−31,8
500	10	0,16	0,016	−35,9
630	10	0,101	0,0101	−39,9
	Measured		Calculated	

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Table 3. Example of how to lay out the results of a frequency response measurement

Manual Phase Response Measurement

Some kinds of servo components have more phase shift than would be predicted from the slope of the frequency response curve, because of the effects of pure time delays. Since such components would result in a lower margin of stability, when they are used in a loop, than minimum-phase components, it is sometimes necessary to measure the phase response as a function of frequency (the term "phase response" alone is very often taken to imply frequency as the independent variable). This may be done most simply using an oscilloscope, and the arrangement shown in Fig.5.

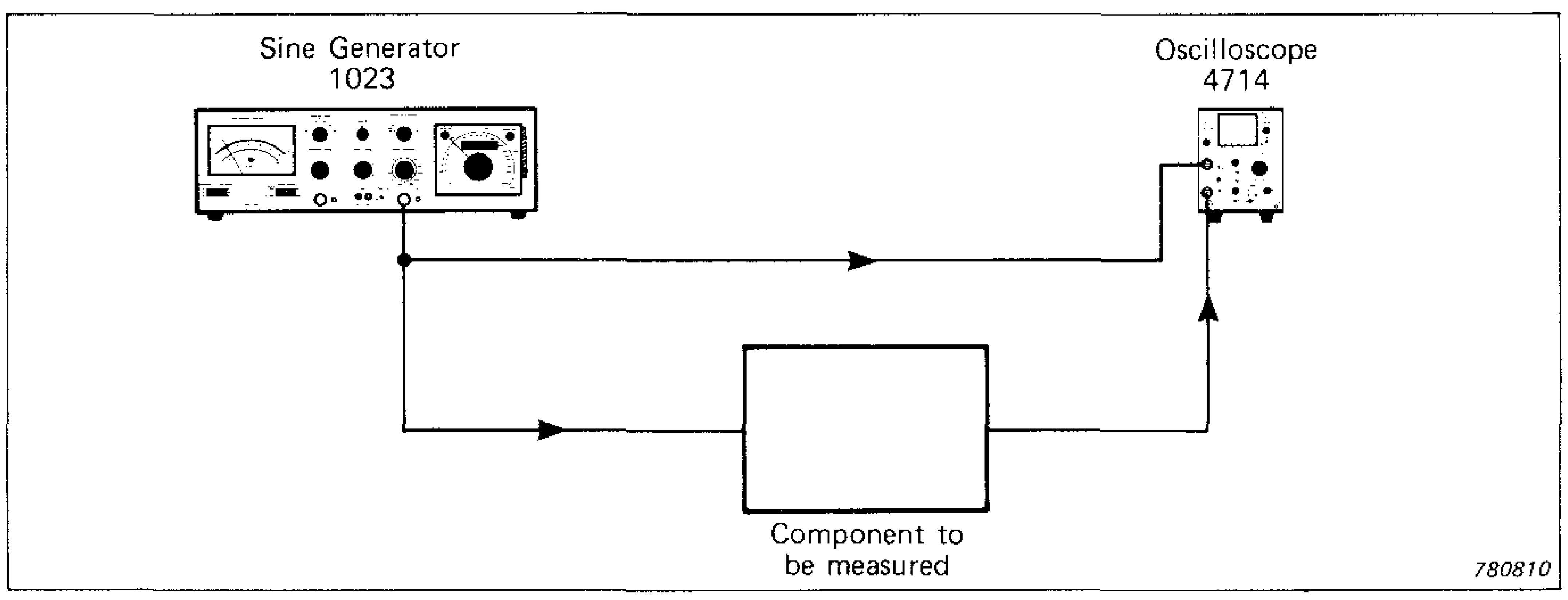

Fig.5. Arrangement of equipment for measuring the phase response of a control loop component or subsystem

If the oscilloscope is provided with an X-input, the signal at the component input may be fed to the X plates and the signal at the output to the Y plates. The resulting Lissajous figure display and the method of calculating gain and phase are shown in Fig.6. This for-

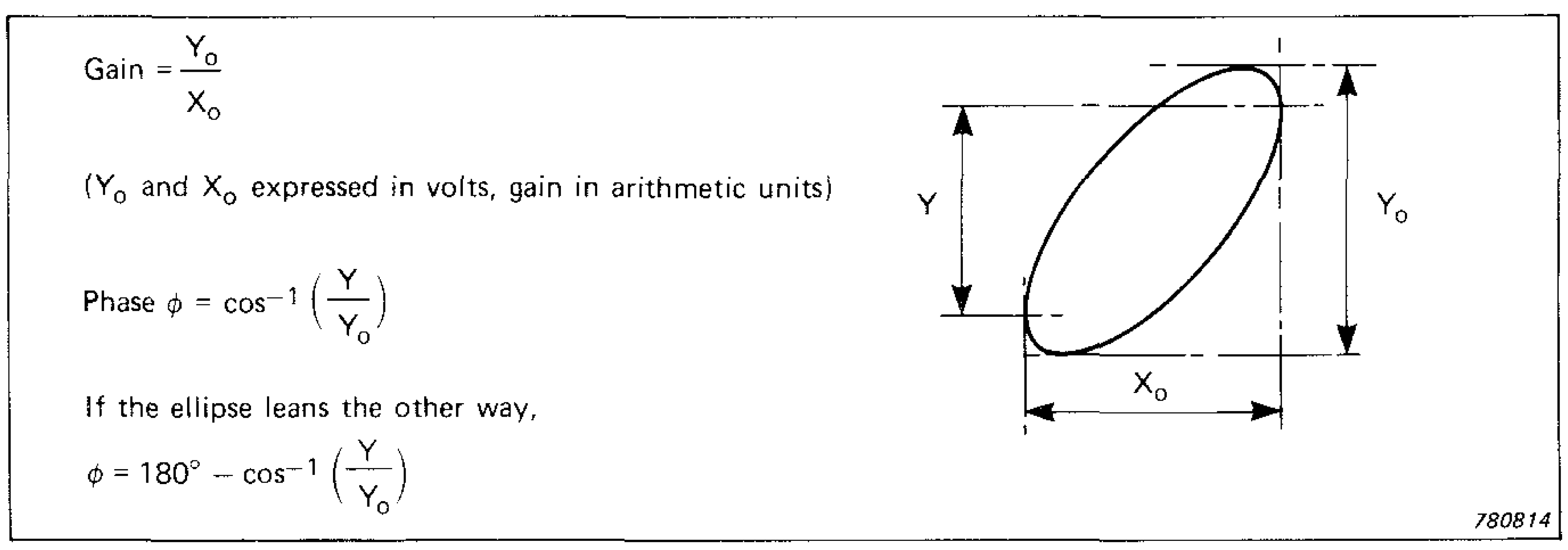

Fig.6. Method of obtaining gain and phase response of a component or subsystem, using an X-Y oscilloscope display

mat is not obtainable with the Type 4714 Oscilloscope shown in Fig.5, for which the methods described below are suitable. However, the X-Y method may be the only one possible with some service oscilloscopes.

If the oscilloscope has an external trigger facility, the sinewave at the input of the component to be measured may be used to trigger the timebase at the 0° phase. Make this adjustment by temporarily feeding the same input sinewave to the Y input of the oscilloscope and adjusting the trigger to obtain a display as in Fig.7(a). This normally needs to be done at only one frequency although it is wise to check that the trigger voltage is not affected by frequency. When the oscilloscope is reconnected to display the output sinewave, the phase can be determined as shown in Fig.7(b).

If a double-beam oscilloscope is available, the output sinewave should be connected to the Y_1 amplifier of the oscilloscope, and the input sinewave to the Y_2 amplifier. This will give a display typically similar to Fig.8, and phase and gain may be determined as indicated.

Since these measurements have to be repeated at a number of frequencies to obtain a plot, it is a fairly time-consuming task and not recommended for other than occasional response measurements. The phase response may be obtained much more easily, and accurately, if a phase meter is available, using the arrangement shown in Fig.9.

With this arrangement the phase may be read directly at each frequency. The gain may be determined by including an AC voltmeter as in Fig.4.

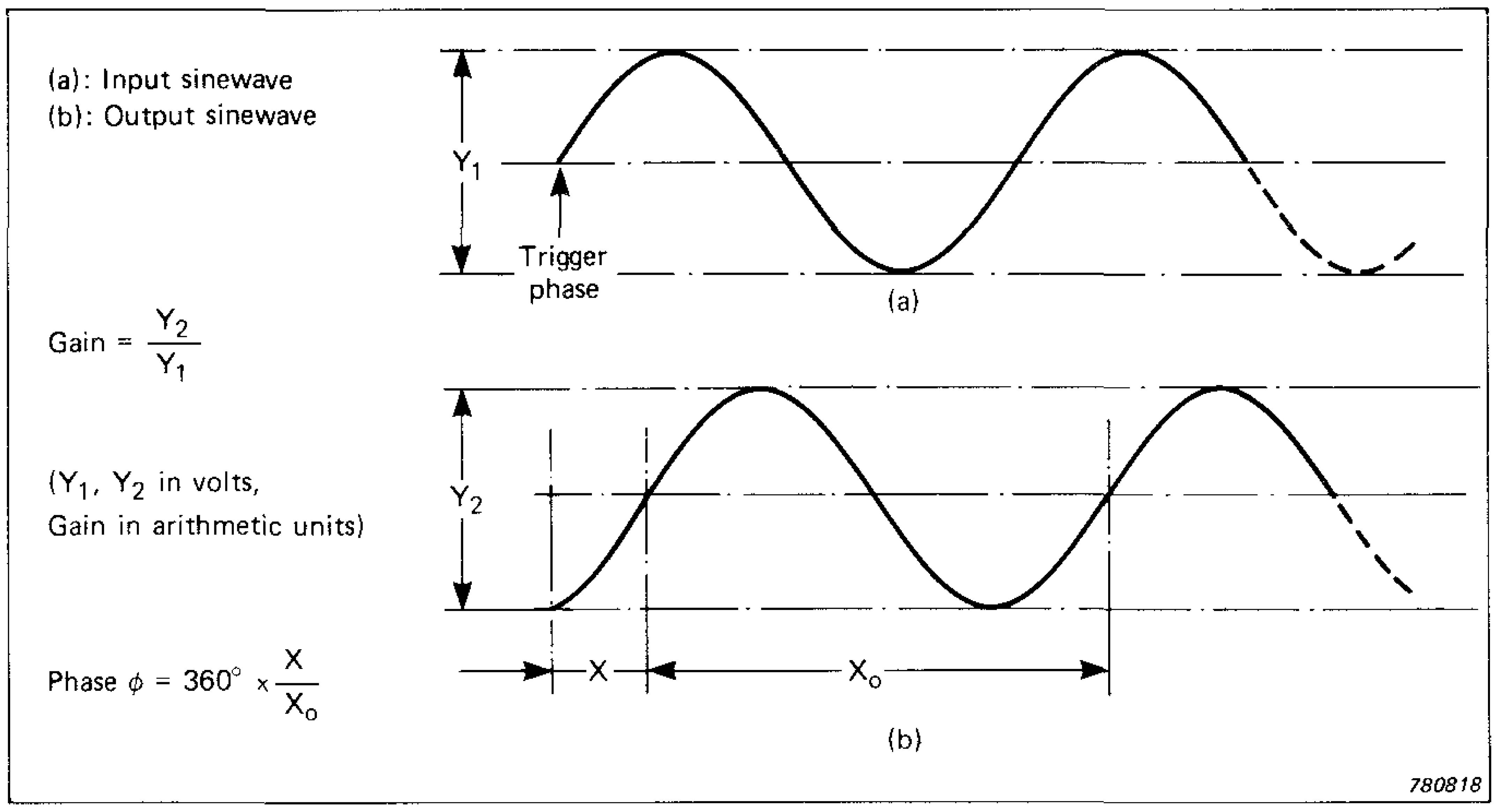

Fig.7. Method of obtaining gain and phase response of a component or subsystem, using a triggered oscilloscope display

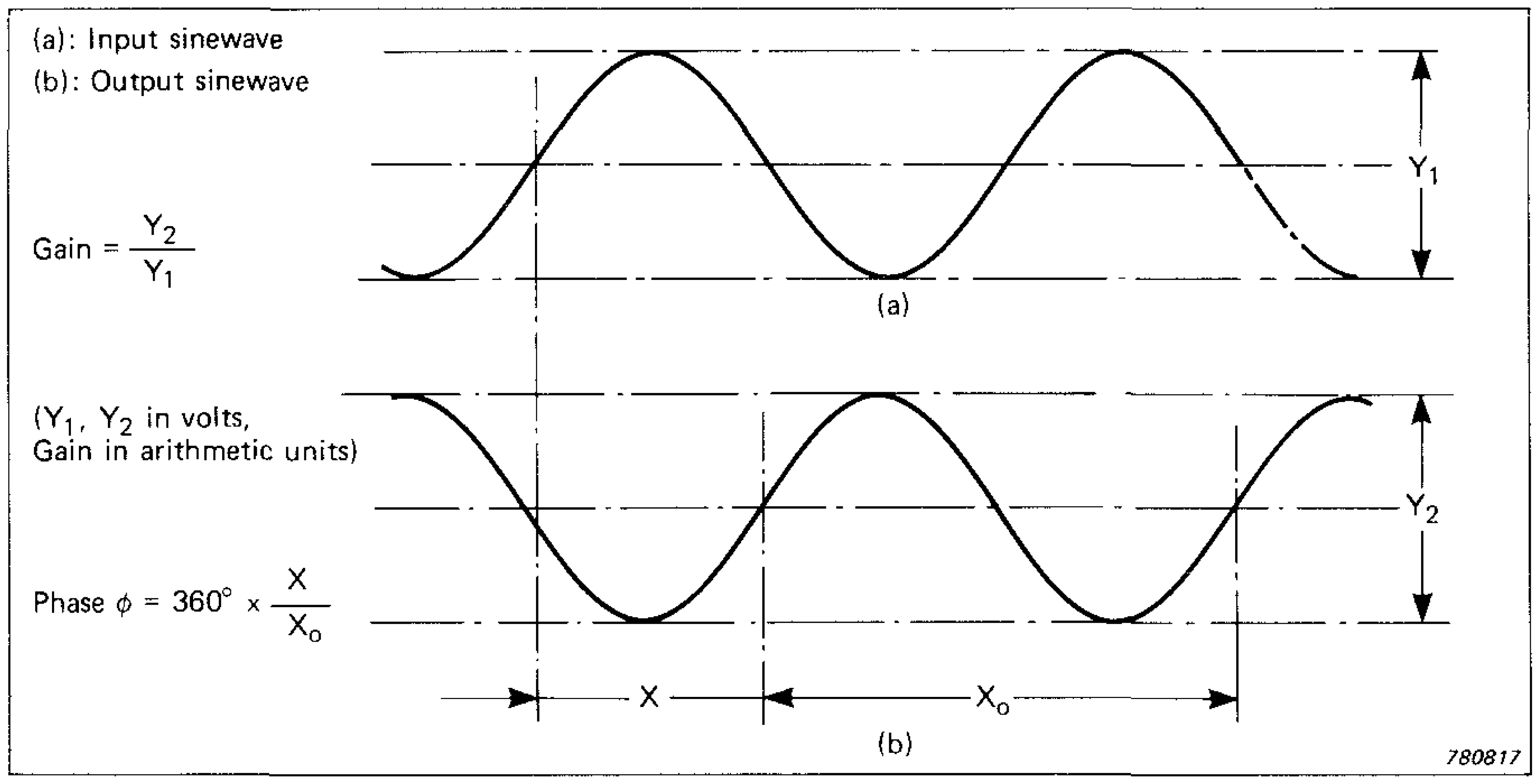

Fig.8. Method of obtaining gain and phase response of a component or subsystem, using a double-beam oscilloscope

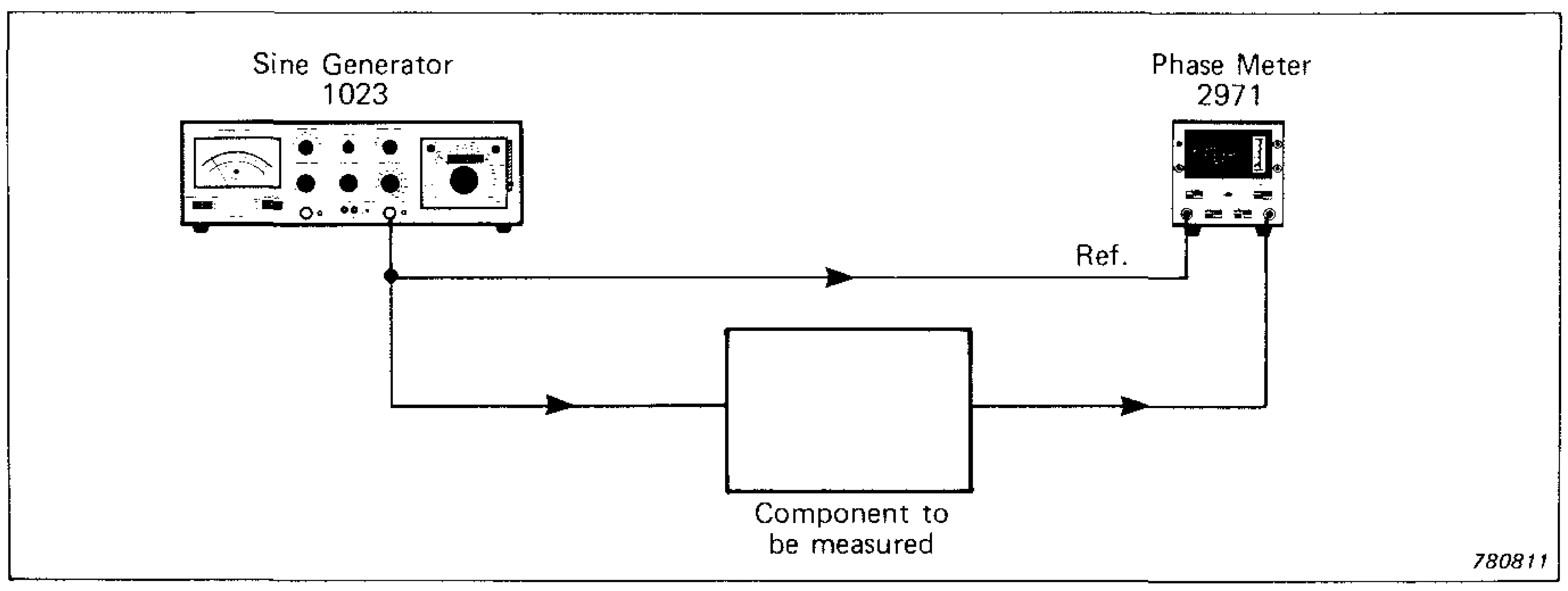

Fig.9. Arrangements of equipment for measuring phase response of a component or subsystem

Automatic Response Measurement

Where many points have to be plotted, or repeated plots to be produced, the above methods become very time-consuming and it is an advantage to employ a sine generator whose frequency may be swept automatically in synchronism with the sweep of a graphic recorder loaded with frequency-graduated paper.

It is very desirable that the frequency sweep be logarithmic and the synchronization accurate. Figs.10 and 11 show examples of arrangements of equipment suitable for producing Bode Plots automatically, and Fig.12 shows an example of such a plot, made with the arrangement of Fig.11.

The graphic level recorder shown in Fig.10 includes a signal rectifier and a single pen servo system having a logarithmic potentiometer, resulting in a pen deflection directly proportional to the component gain, in dB. The frequency-graduated chart paper is contained on a roll which is fed past the pen in synchronism with the frequency sweep of the sine generator.

In Fig.11 a measuring amplifier is included to provide a DC output proportional to the logarithm of the sinewave amplitude (and thus of the component gain). This DC output controls the Y-deflection of the X-Y recorder, which has two pen servos. The X deflection is controlled by a built-in sweep generator which also controls the logarithmic frequency sweep of the sine generator.

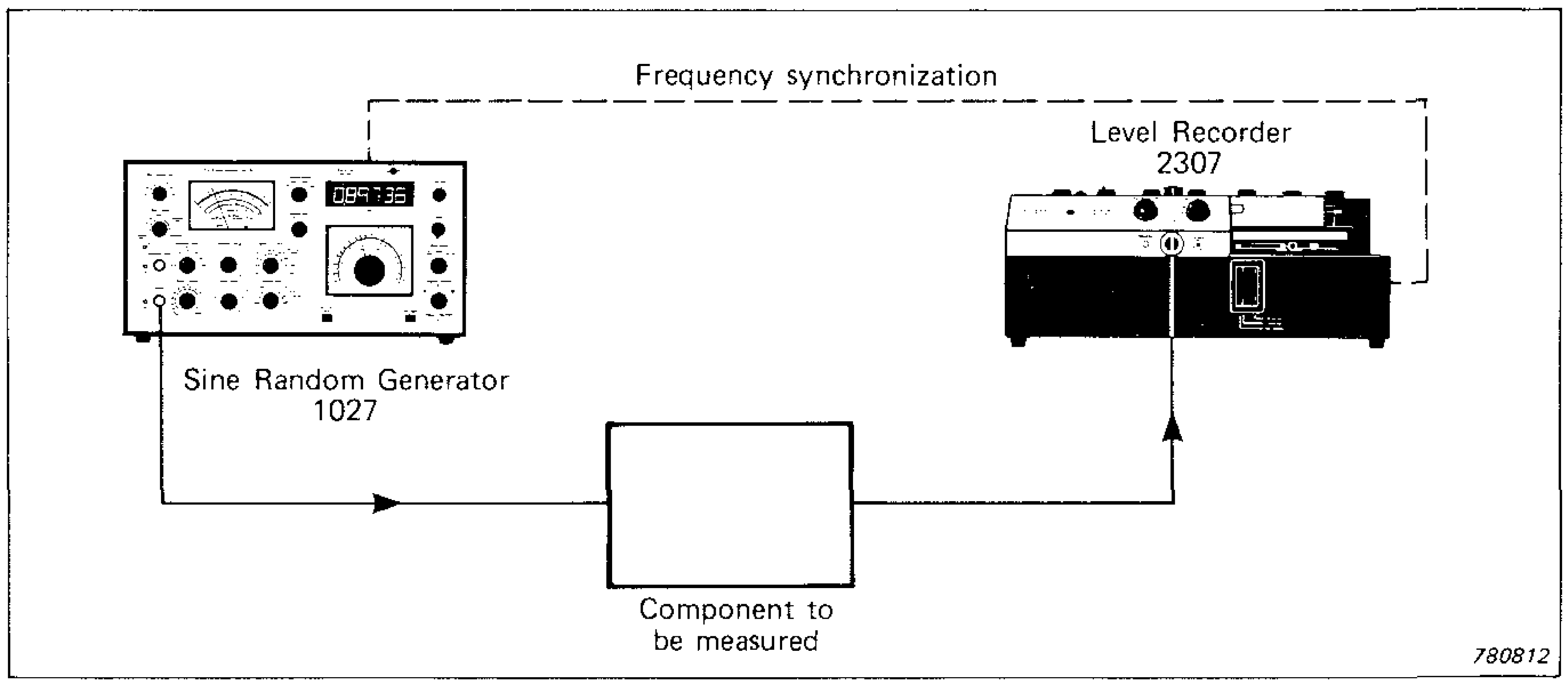

Fig.10. Arrangement of equipment for recording automatic Bode plots of a control component or subsystem

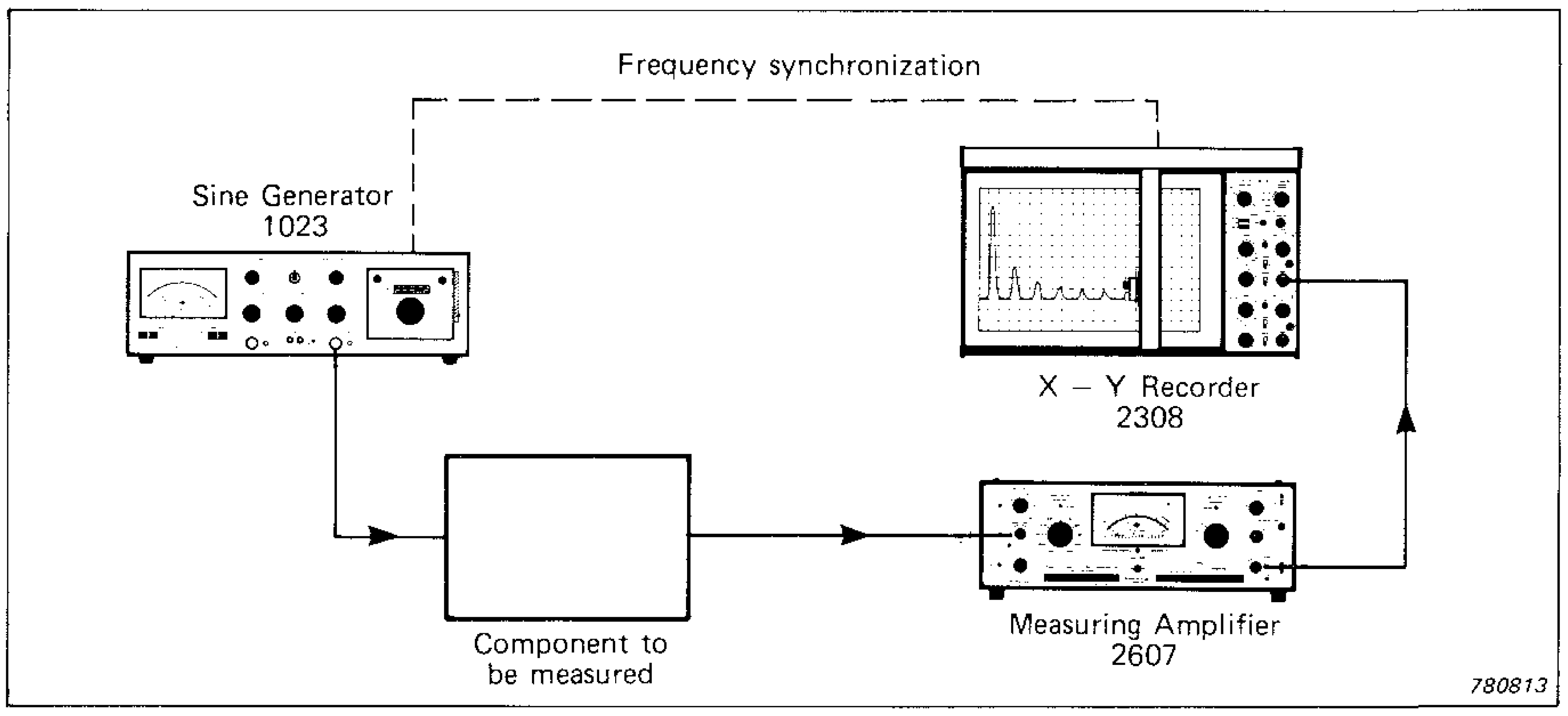

Fig.11. Arrangement of equipment for recording automatic Bode plots of a control component or subsystem

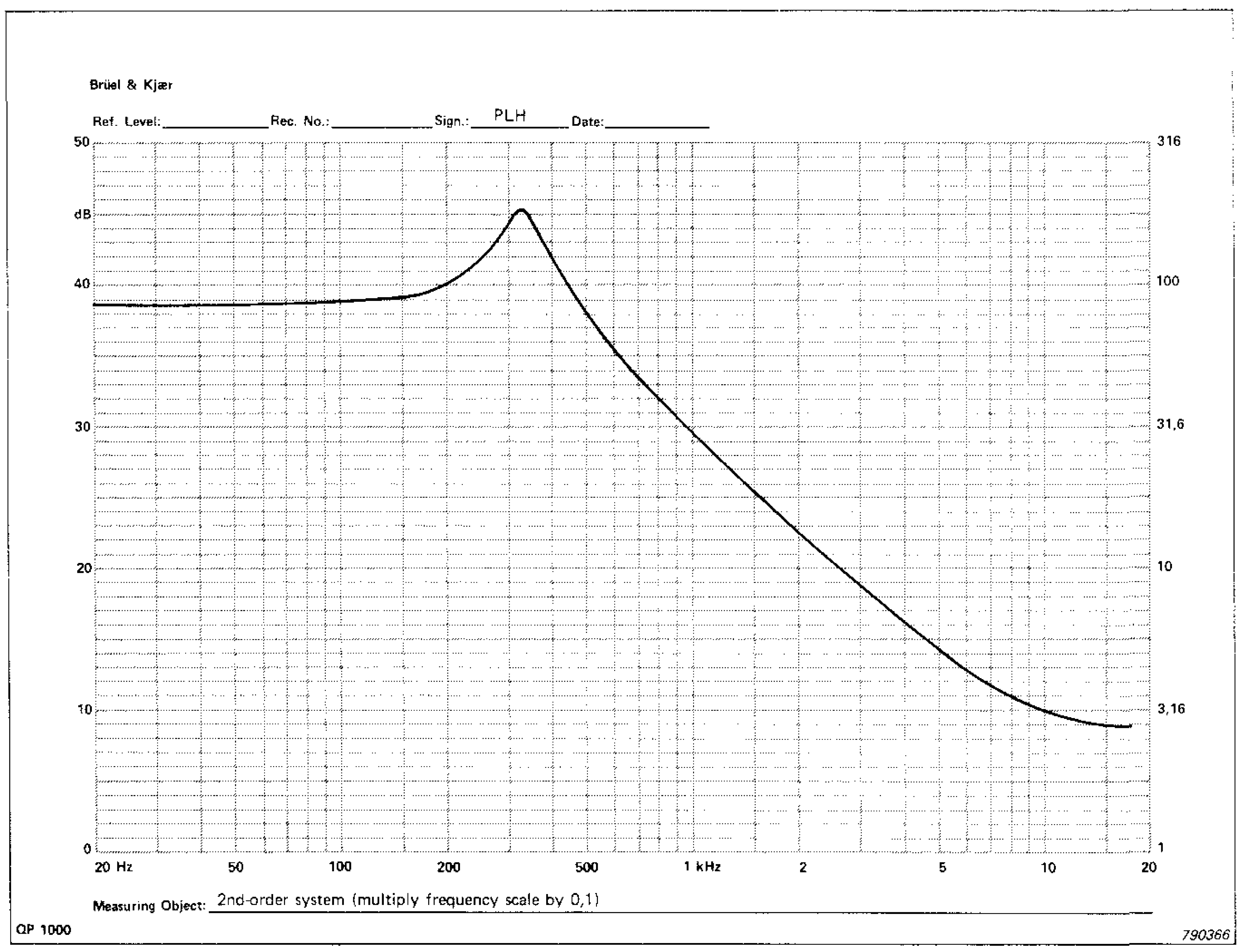

Fig.12. Example of automatic Bode plot

Automatic plots of gain and phase against frequency may be made if a two-channel graphic recorder is available. The arrangement shown in Fig. 13 is particularly convenient as the two-channel recorder performs rectification and lin.-log. conversion of the sinewave signal, and the phase meter produces a DC output proportional to phase.

Examples of automatic plots of gain and phase as functions of frequency are shown in Figs. 2 and 14.

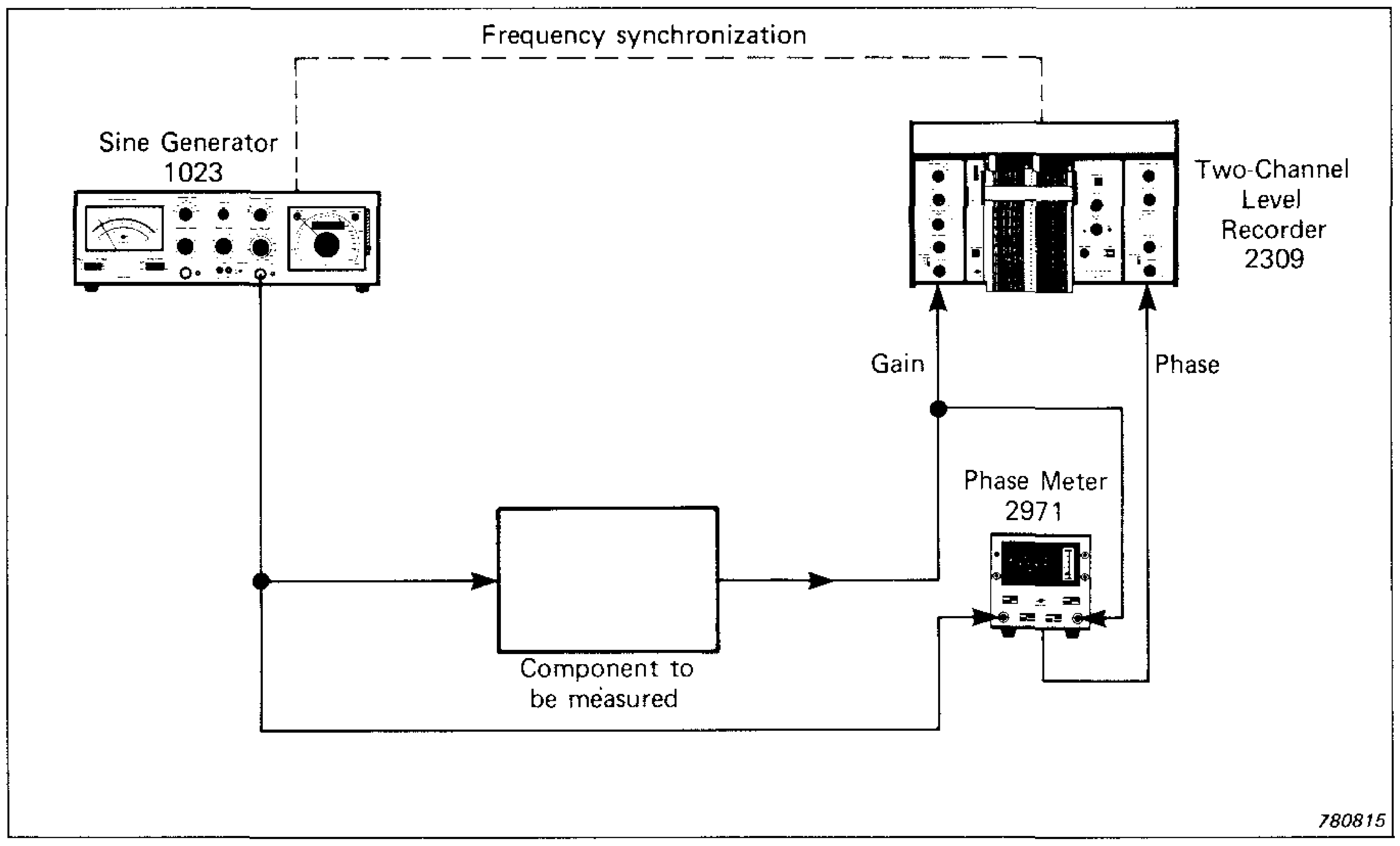

Fig. 13. Arrangement of equipment for recording automatic plots of gain and phase as functions of frequency

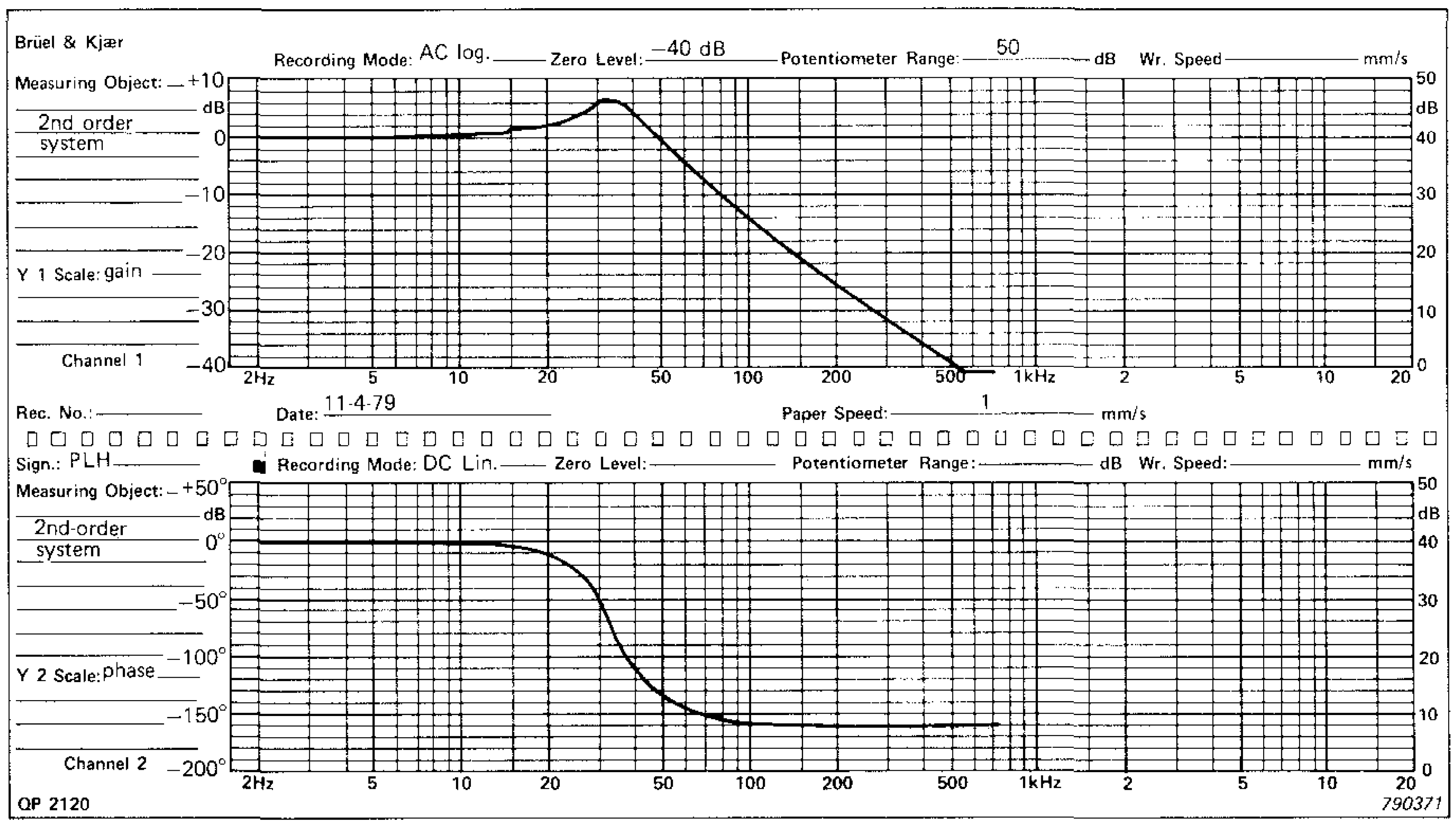

Fig. 14. Example of automatic plot of gain and phase

Response Measurement of Noisy Components and Subsystems

The active components and subsystems used in control loops, such as amplifiers, often generate noise — that is, unwanted perturbations — whose amplitude at the output may exceed the sinewave used for response measurement at some frequencies. This is especially so of components used in systems embodying carriers or subcarriers, such as phase-locked loops and monitoring devices in power control systems. The phase detector used in a phase-locked loop will generate some ripple at twice the carrier frequency, for example. A current transformer used to monitor the current in a large electrical machine connected to the AC mains will generate ripple at 100 Hz or 120 Hz (de-

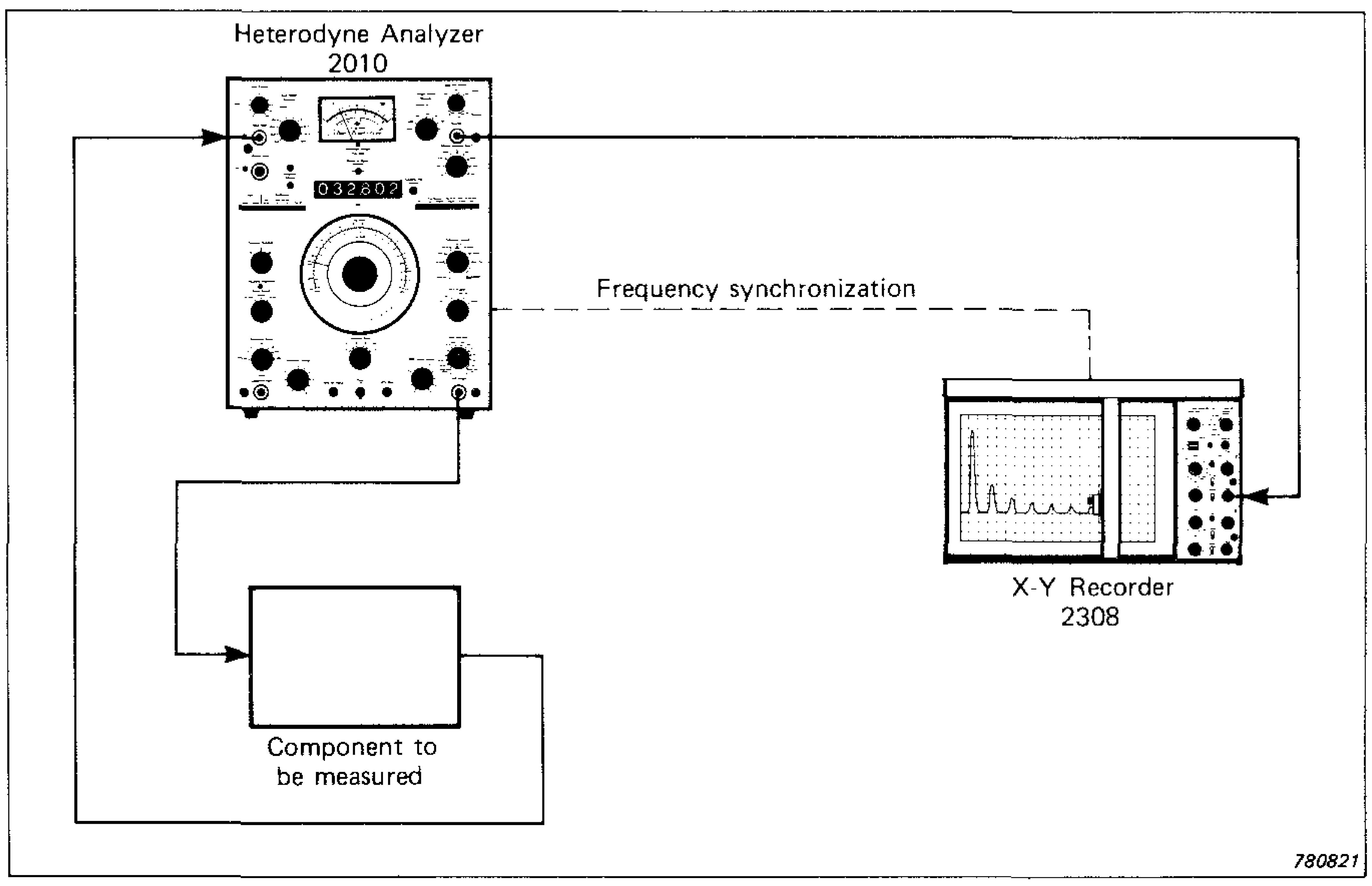

Fig. 15. Arrangement of equipment for recording frequency response of noisy component or subsystem

pending on whether mains frequency is 50 Hz or 60 Hz).

To make measurements of frequency response on components and subsystems under these circumstances calls for a band-pass filter in the output measuring circuit. This filter must reject all frequencies except those within a narrow band centred on the frequency of the generator. It therefore has to be designed to sweep its centre frequency in synchronism with the generator, preferably automatically.

The Heterodyne Analyzer used in the arrangement of equipment shown in Fig.15 incorporates a filter meeting these requirements. It also has built in a sine generator (with which it is synchronized) and a measuring amplifier. This amplifier can be set to provide a DC logarithmic output for recording (in dB) by the X-Y Recorder. An example of the benefit of this arrangement is shown in Fig.16 and 17, which show automatic frequency response recordings of a noisy 2nd-order system with and without selective filtering.

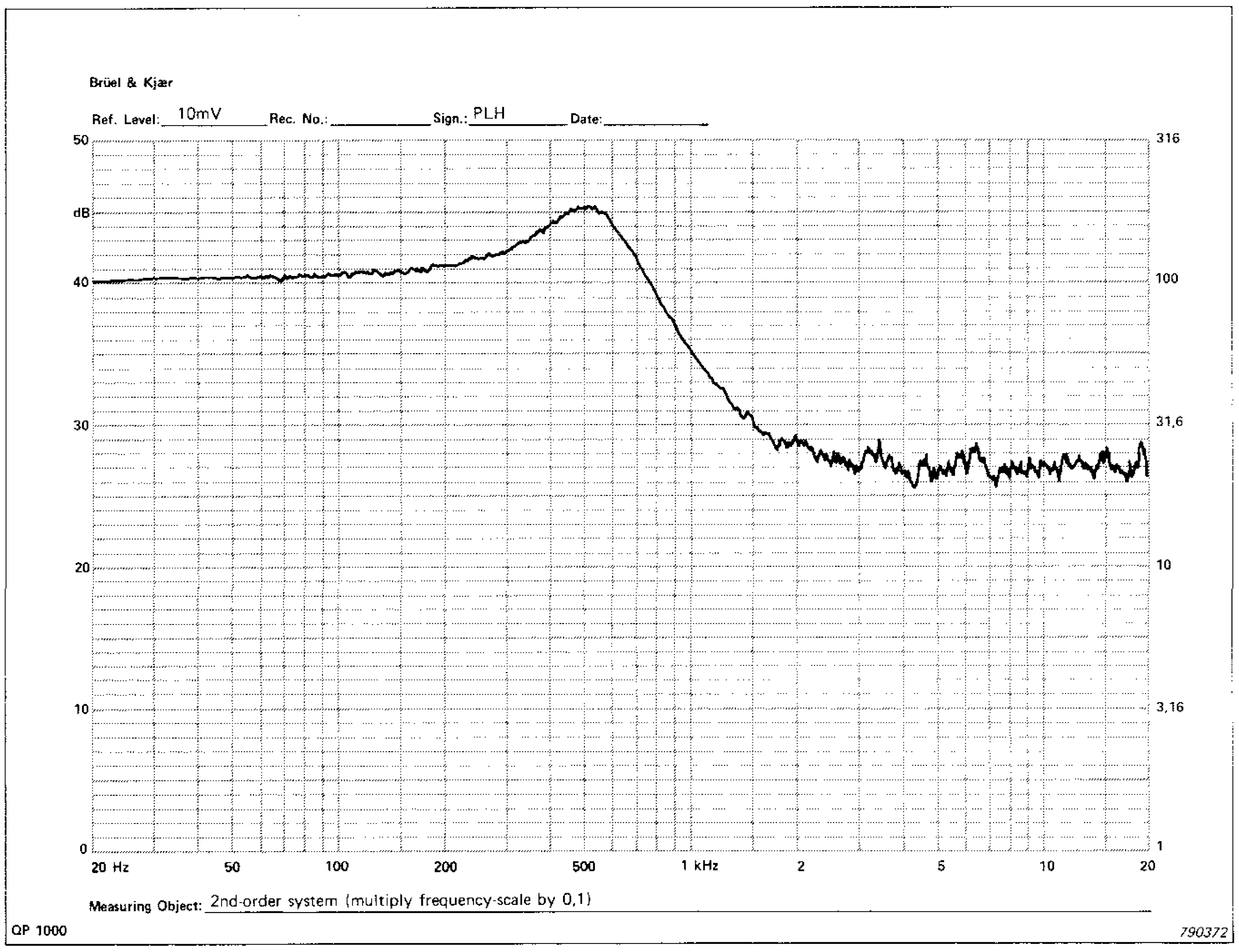

Fig.16. **Example of an automatic non-selective frequency response measurement on a noisy system**

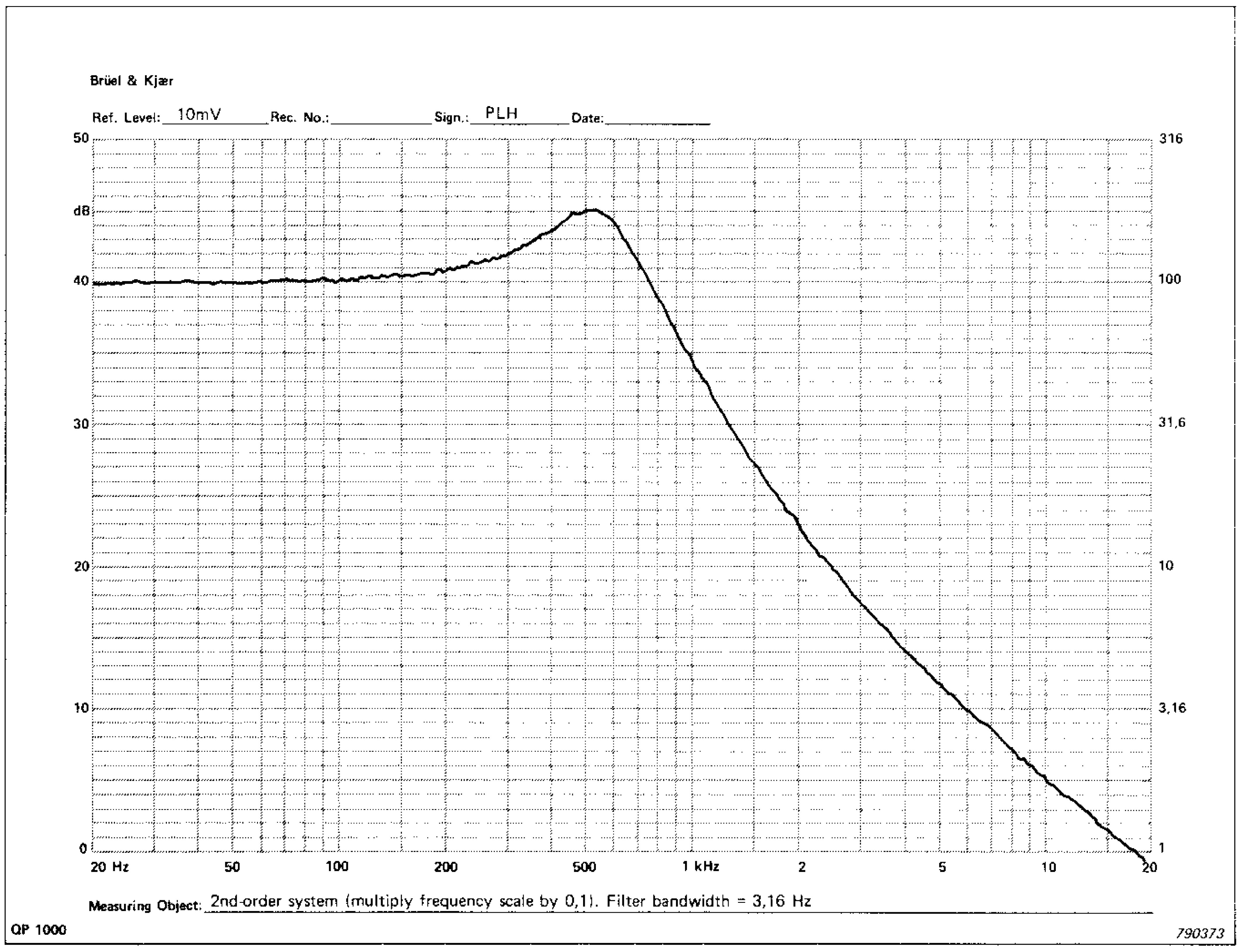

Fig.17. **The same measurement as in Fig.16, made using the selective filtering of a heterodyne analyzer. Note how the noise no longer interferes with the measurement**

Component Response Measurement under Closed-Loop Conditions

It is sometimes required to deduce the transfer characteristic of a part of a loop without feeding a signal directly into the input port of the portion of the loop to be measured. If the loop is closed, then a signal fed into the loop input will appear at every point of the loop, thereby satisfying this requirement. Fig.18 illustrates connections for such a measurement.

However, this is a fairly tedious measurement as both the input and output voltages on the component or subsystem to be measured will often vary with frequency, so a calculation has to be made at each frequency of measurement to determine the gain of the component.

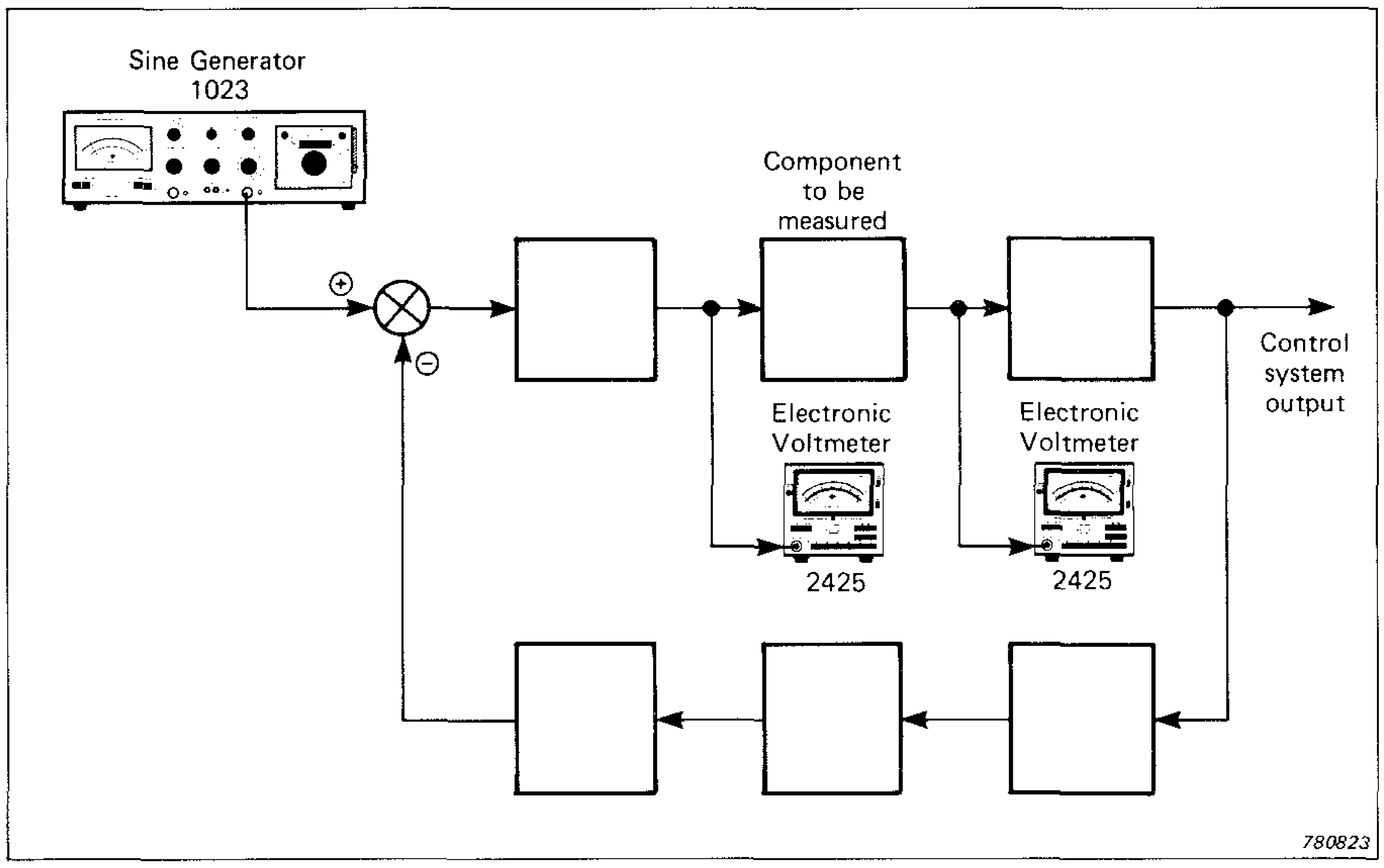

Fig.18. Component transfer characteristic measurement without breaking loop — manual method

The measurement can be automated, and a chart recording made of the actual transfer characteristic, by using a sine generator equipped with a compressor. A compressor, in this context, is an electronic amplitude control for regulating a level at some point in a system other than the sine generator output terminal. A feed from the point where regulation is required is taken to the compressor input of the sine generator, and the compressor electronically adjusts the generator output voltage to obtain a constant, preset voltage at its input. This voltage is proportional to the voltage at the input (of the component or subsystem to be measured), which will also, therefore, be kept constant.

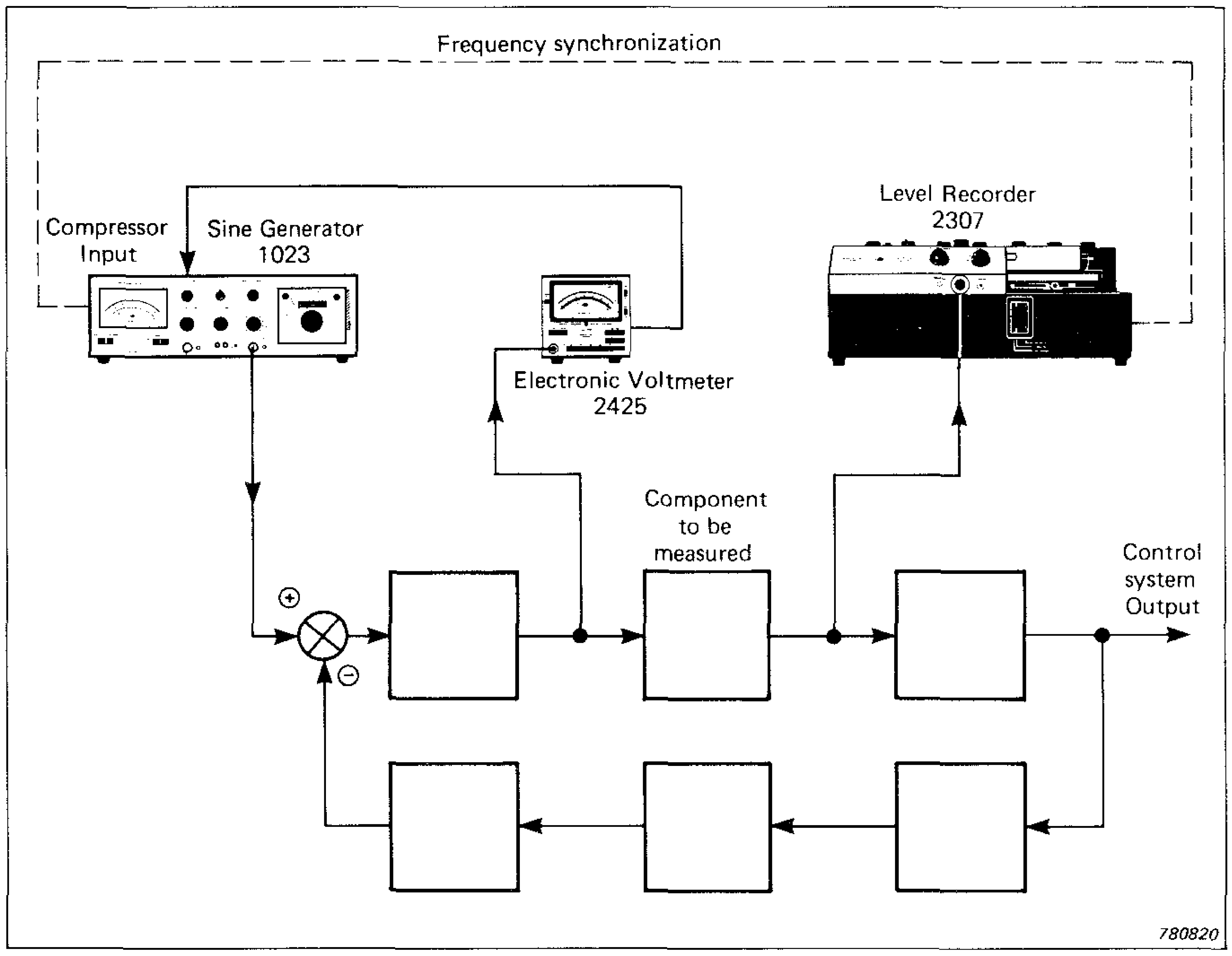

Fig.19. Component transfer characteristic measurement without breaking loop — automatic method

Fig.19 shows this principle implemented in the measurement situation of Fig.18. The measuring amplifier serves to provide a means of amplifying the voltage at the component input to a level within the compressor input range. This voltage is kept constant by the compressor, so the voltage recorded at the component or subsystem output by the level recorder is proportional to the transfer coefficient of the component at each frequency.

Since there are now two loops interacting, it is important to check that the specimen loop does not cause instability in the compressor loop, and vice versa.

Response Measurement of Mechanical and Electromechanical Components

With all the measurement methods treated above it has been assumed that both input and output ports of the component to be measured are electrical. This is often true even for electromechanical components. For example, a DC electric motor to be used in a torque-controlled drive has a mechanical power output, but if the servo loop operates by monitoring motor current then its output port for control purposes is the motor current and its input port the motor voltage.

However, there are many servo loops where either an excitation signal or an output signal, or both, is, or are, mechanical.

It is still true that in general the most useful measurement and analysis techniques are by sinusoidal frequency response, and the above methods may be used with the aid of suitable linear transducers. A transducer is a device which converts power from one pair of flow variables (voltage, current, for example) to another (velocity, force, for example).

To measure sinusoidal acceleration, velocity or displacement at the output port of a mechanical component, a piezoelectric accelerometer may be used. These transducers are characterized by small size, robust construction, low electrical noise, excellent linearity, wide-range uniform frequency response and passive (i.e., non-amplifying) operation. Since it abstracts negligible power from the measurement specimen, the piezoelectric accelerometer used should be connected to a suitable conditioning preamplifier to provide correct matching to measuring instrumentation. The conditioning preamplifier will normally incorporate switchable integrating networks to select output proportional to acceleration, velocity or displacement.

An example of a manual arrangement for measuring the frequency response of a component with an electrical input and a mechanical output is shown in Fig.20.

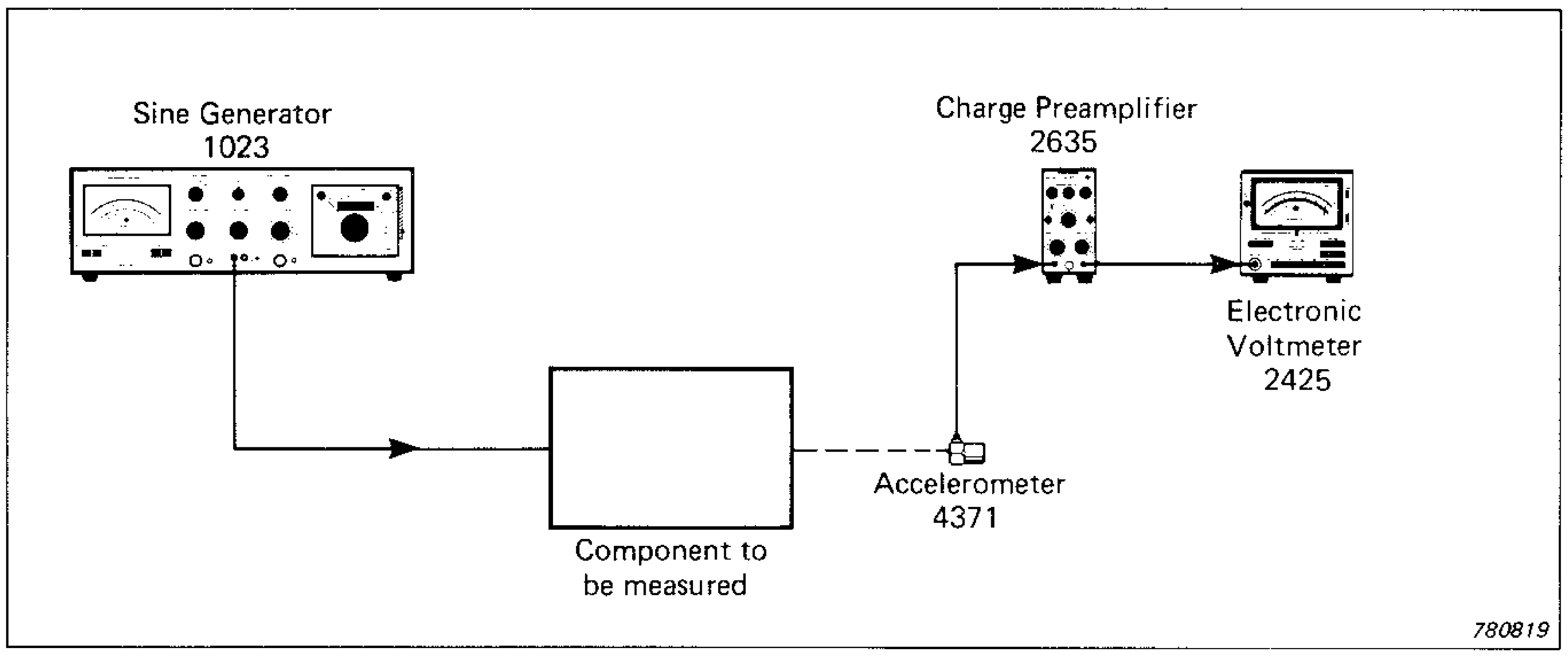

Fig.20. Frequency response measurement on a component or subsystem having a mechanical output

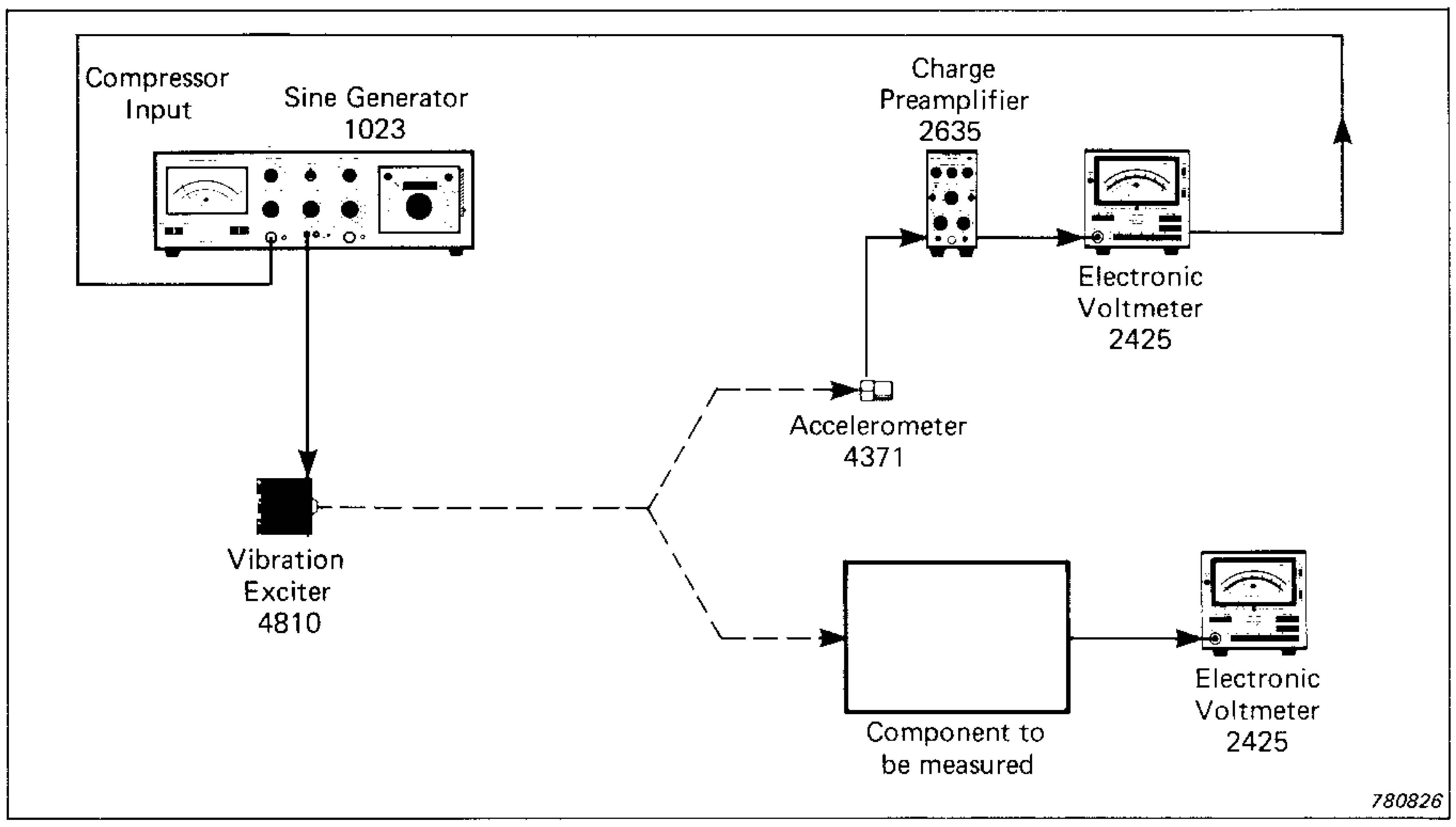

Fig.21. Frequency response measurement on a component or subsystem having a mechanical input

In the case of a component for which it is the input which is mechanical, a suitable transducer for sinewave measurement purposes is an electromagnetic vibration exciter. This type of device converts an AC electrical input into a corresponding mechanical accelerating force which is approximately constant over about two decades of frequency (100:1 frequency range). In order to extend the frequency range and render its transfer function sensibly constant and independent of frequency, it is desirable to control the exciter amplitude by means of a piezoelectric accelerometer connected in a compressor loop. This same arrangement may be used to keep the velocity or displacement (instead of the force/acceleration) constant.

The exciter should be mechanically coupled to the component to be tested, and the control accelerometer securely attached to the coupling so as to monitor faithfully the amplitude and phase of the motion. Fig.21 shows an example of such an arrangement, used to measure the frequency response of a servo component having a mechanical input and an electrical output.

Non-Sinusoidal Excitation

Some large industrial components forming parts of control systems do not lend themselves readily to the injection of sinewaves from a sine generator for response testing, for one or more of the following reasons:

1) unacceptable resonances may be excited by the application of a sinewave;

2) there may be difficulties in coupling the generator to the component;

3) the power required to produce sufficient amplitude of oscillation to obtain usable measurements or overcome backlash or 'stiction' may be excessive;

4) deliberate introduction of oscillation may be feared hazardous;

5) the response may be too slow for sinewave measurement.

In such circumstances the only way to measure the frequency response is to allow the component to be excited by whatever perturbations are permitted and to perform frequency analysis of the signals at the input and output ports. In many cases, the permitted perturbations will contain energy over a sufficiently wide range of frequencies to make possible quite useful deduction of the frequency response. However, the measurements must be averaged out over a long time so as to eliminate random interference.

A very useful signal for these purposes is true random "white" or "pink" noise, which may be obtained from a suitable noise or random generator. An arrangement using such a source is shown in Fig.22. An example of measurement is shown in Figs.23 and 24. This example was made using automatic synchronization of the filter shown with a level recorder, but this is not always feasible if it is necessary to use different excitation levels at some frequencies.

The generator illustrated can also generate sinewaves, and narrow-band noise. The response is measured in third-octaves by tuning the filter, and recording the level manually (one-third of an octave = one-tenth of a decade).

This arrangement permits measurements to be made without undue excitation of system resonances, but it suffers many of the other limitations of sinusoidal excitation mentioned above. Normally with large industrial components, measurements on them would be taken in open-loop working conditions, excitation coming from existing working perturbations such as load fluctuations, switch-on transients, etc.

Since the frequency spectrum of the signal at the input cannot be assumed to be flat, it is necessary to perform frequency analysis of

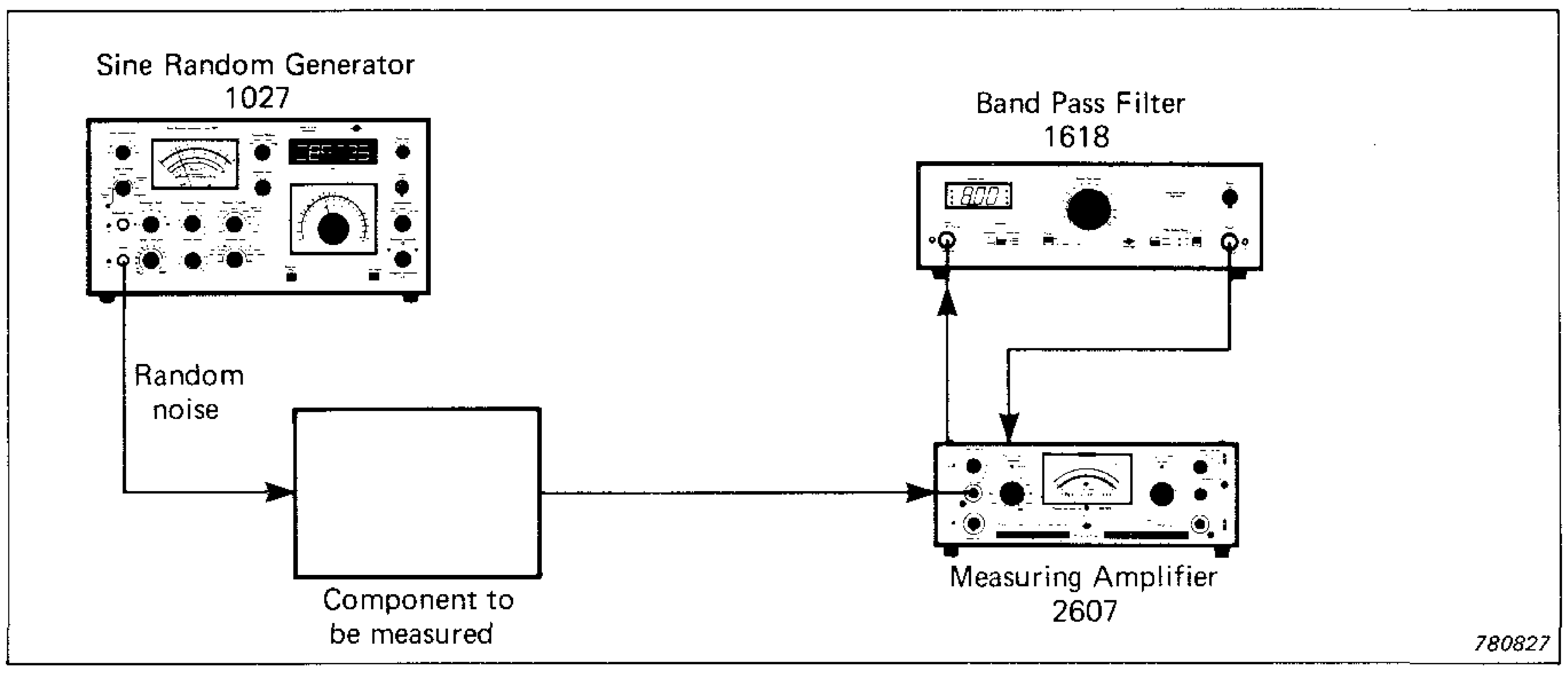

Fig.22. Measurement of component or subsystem frequency response using random noise. Note that where the component input impedance may affect the flatness of the input spectrum, the latter should also be measured

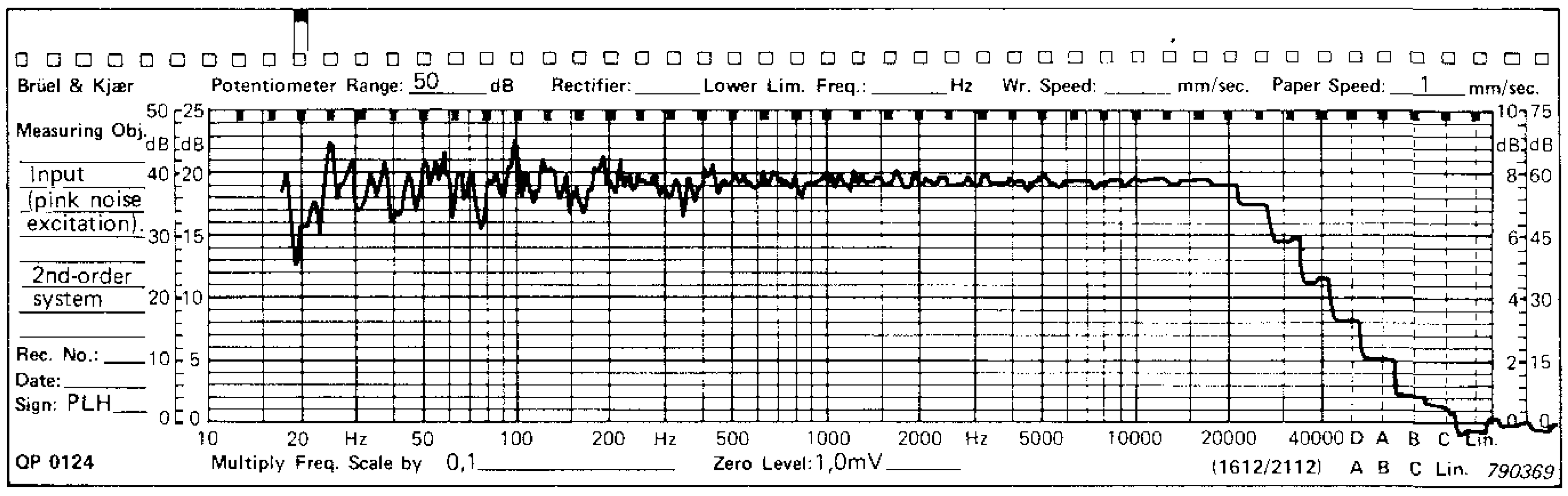

Fig.23. Example of an automatic frequency response measurement, using pink noise as the exciting source, in an arrangement similar to Fig.22. Pink noise spectrum. This is the input spectrum, and it proves to be flat over the frequency range of interest, in this case

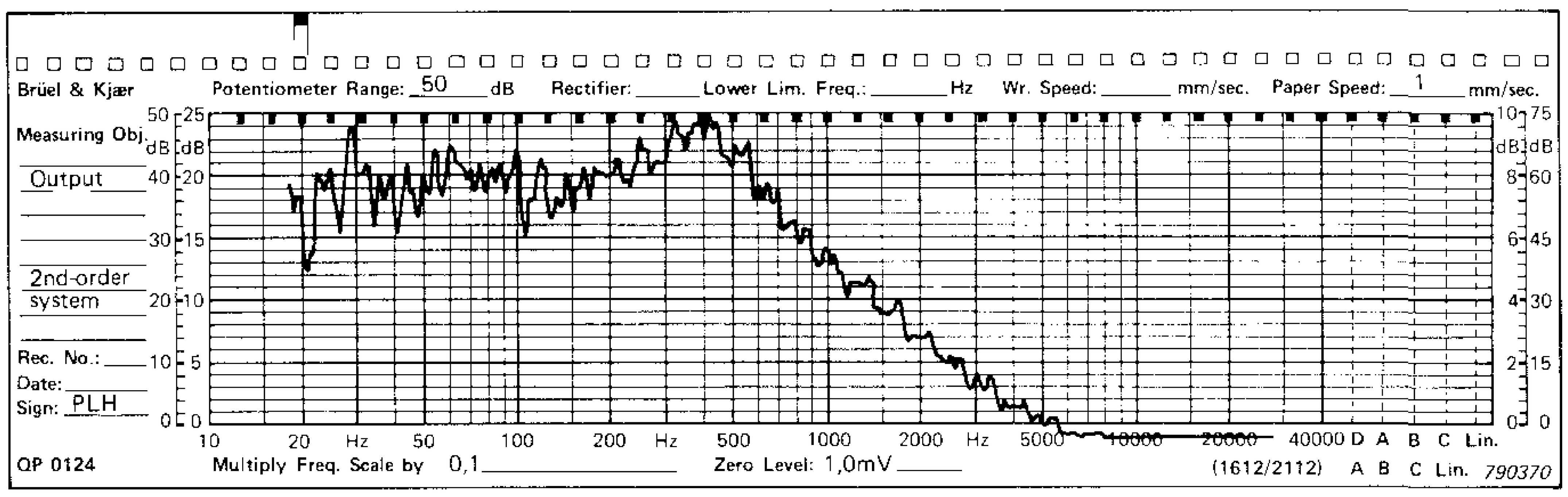

Fig.24. Output spectrum corresponding to Fig.23

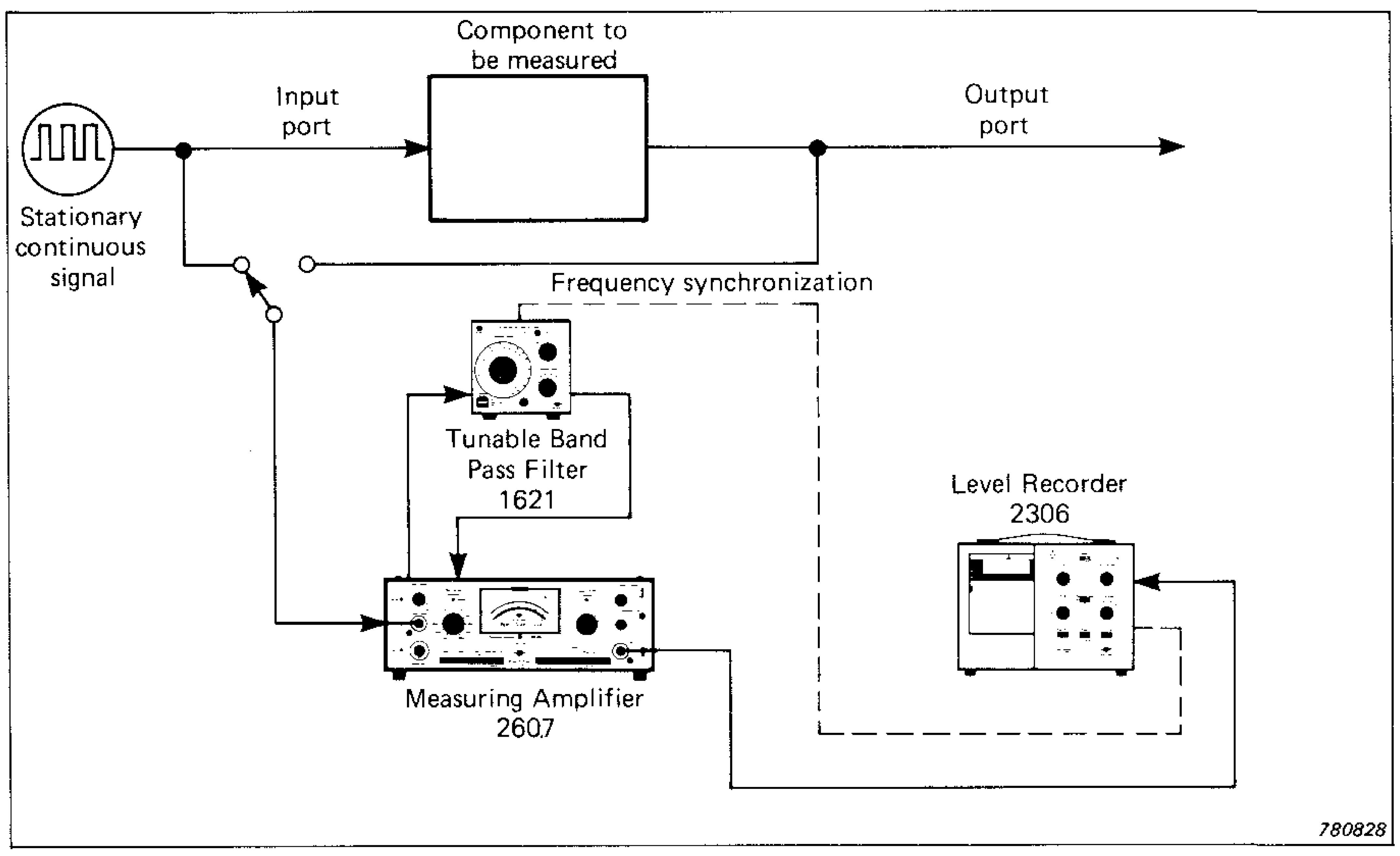

Fig.25. Measurement of component transfer characteristic by frequency analysis of existing stationary signal

both input and output signals. The method of analysis depends very much on the type of signal being analyzed.

Signal types are broadly classified according to whether they are stationary or non-stationary.

Stationary signals are simplest to use for frequency response measurements, but less likely to be encountered in practice. A stationary signal is one whose frequency spectrum is independent of time. In other words, the measured spectrum would be the same if the measurement were repeated after ten minutes, say. If the component to be measured can be excited by a stationary signal which is also expected to contain energy at a wide range of frequencies, then the input and output frequency spectra may be measured in turn, using a bandpass filter whose centre frequency is swept through the frequency range of interest slowly enough to obtain a steady, repeatable reading at each frequency. A suitable arrangement is shown in Fig.25. Provided the frequency analysis is made with levels recorded in dB, the transfer characteristic can be obtained by subtracting the input level from the output level. Sweeping may be manual or automatic. An example of the results of such a measurement is shown in Figs.26 and 27.

If the available input signal is non-stationary, there are two techniques for measurement, both requiring real-time frequency analysis, depending on whether the signal is of a transient or continuous nature.

A transient signal is characteristic of a discrete event such as switch-on, shut-down, sudden change in load etc. Generally speaking a signal is regarded as transient if it is bounded (in time) by periods of zero level, or of a level of insufficient energy for useful measurements. It is in the nature of transients that they very often contain energy fairly broadly distributed through the frequency spectrum, and this renders them useful for response measurement.

In most of the components to be dealt with where transient input sig-

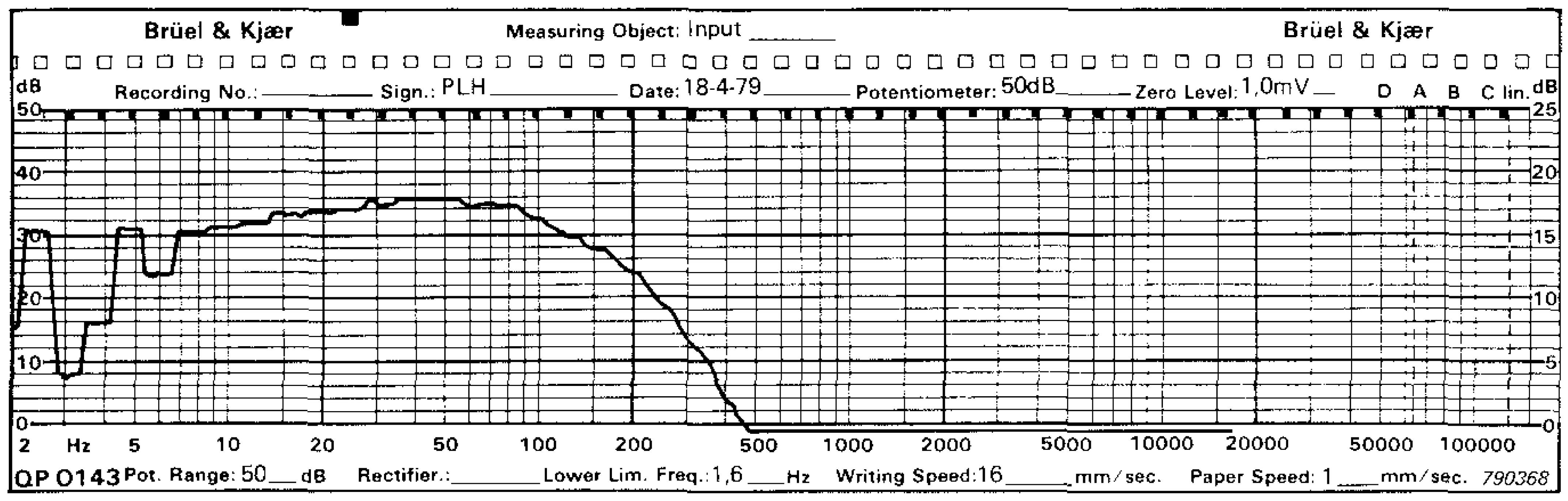

Fig.26. Example of frequency response measurement using a stationary signal, (a 2,4 Hz train of 2 ms pulses) in an arrangement similar to Fig.24 (but using a discrete third-octave band pass filter set). Input spectrum

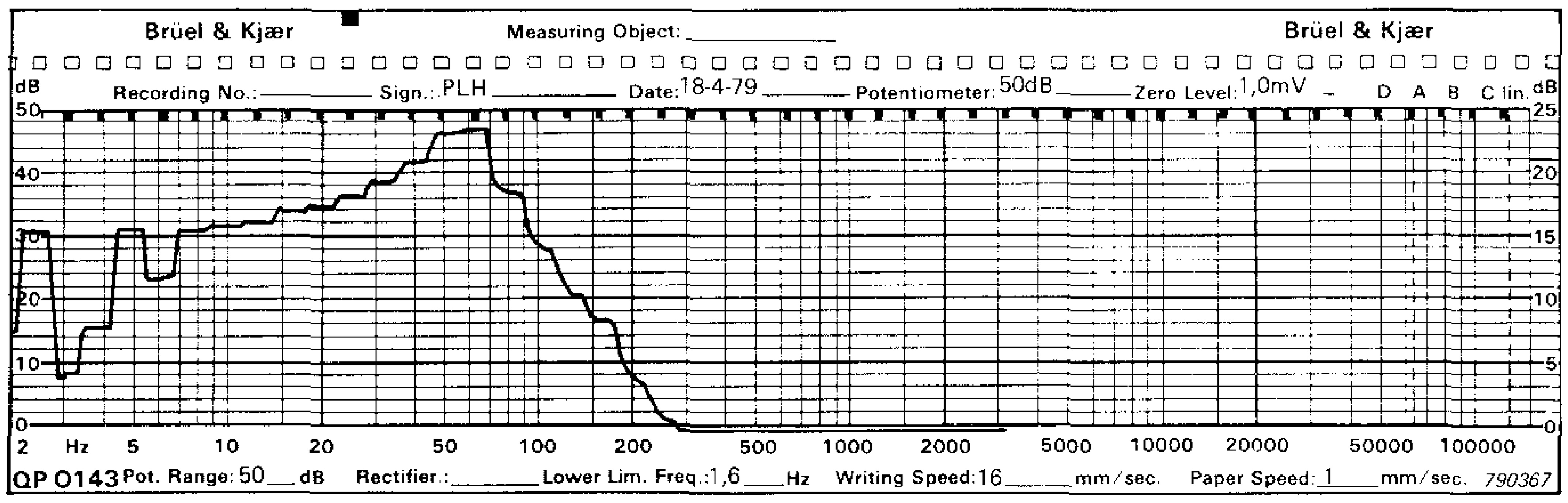

Fig.27. Output spectrum corresponding to Fig.26

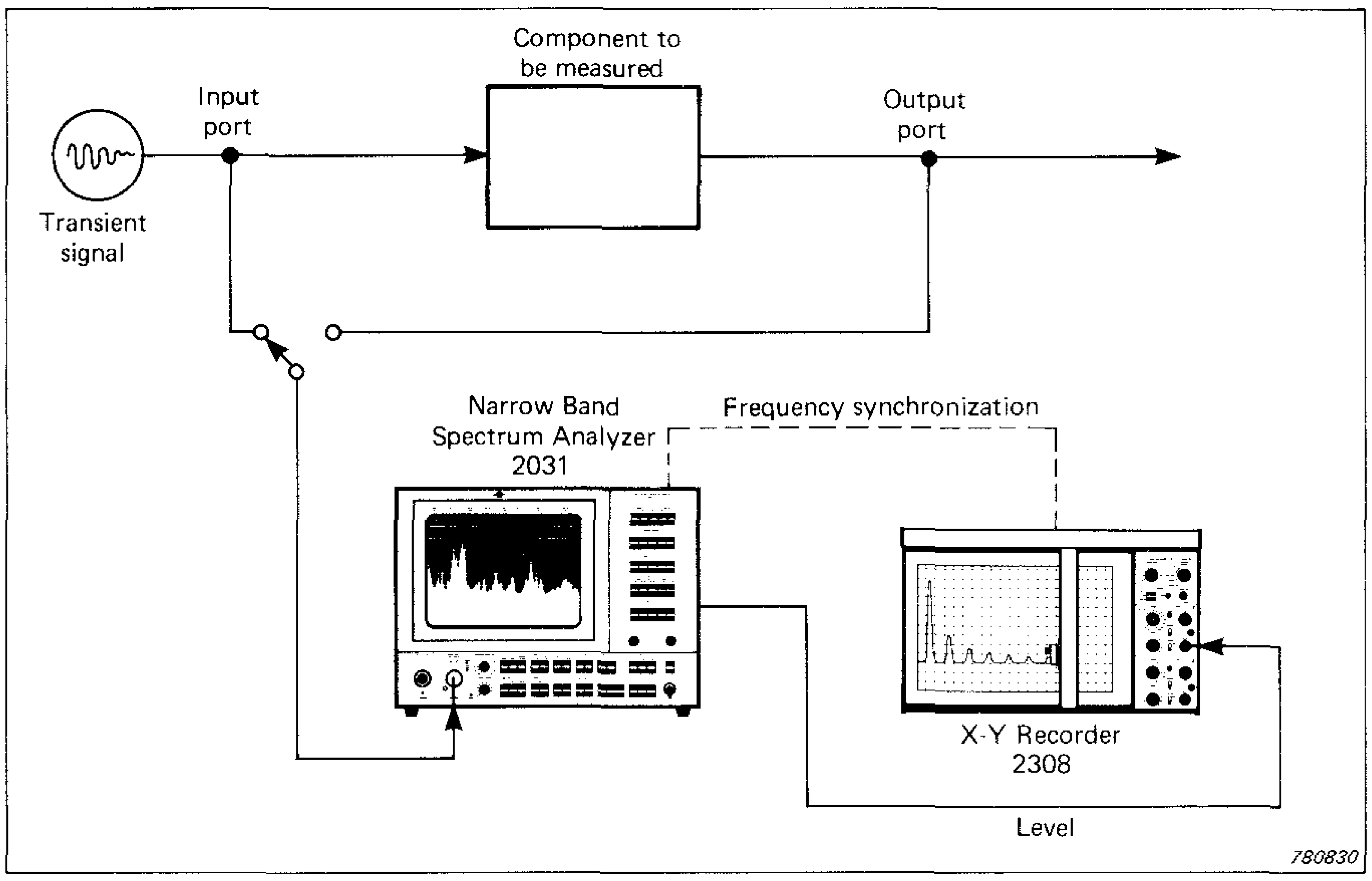

Fig.28. Measurement of component frequency response by frequency analysis of existing transient signals

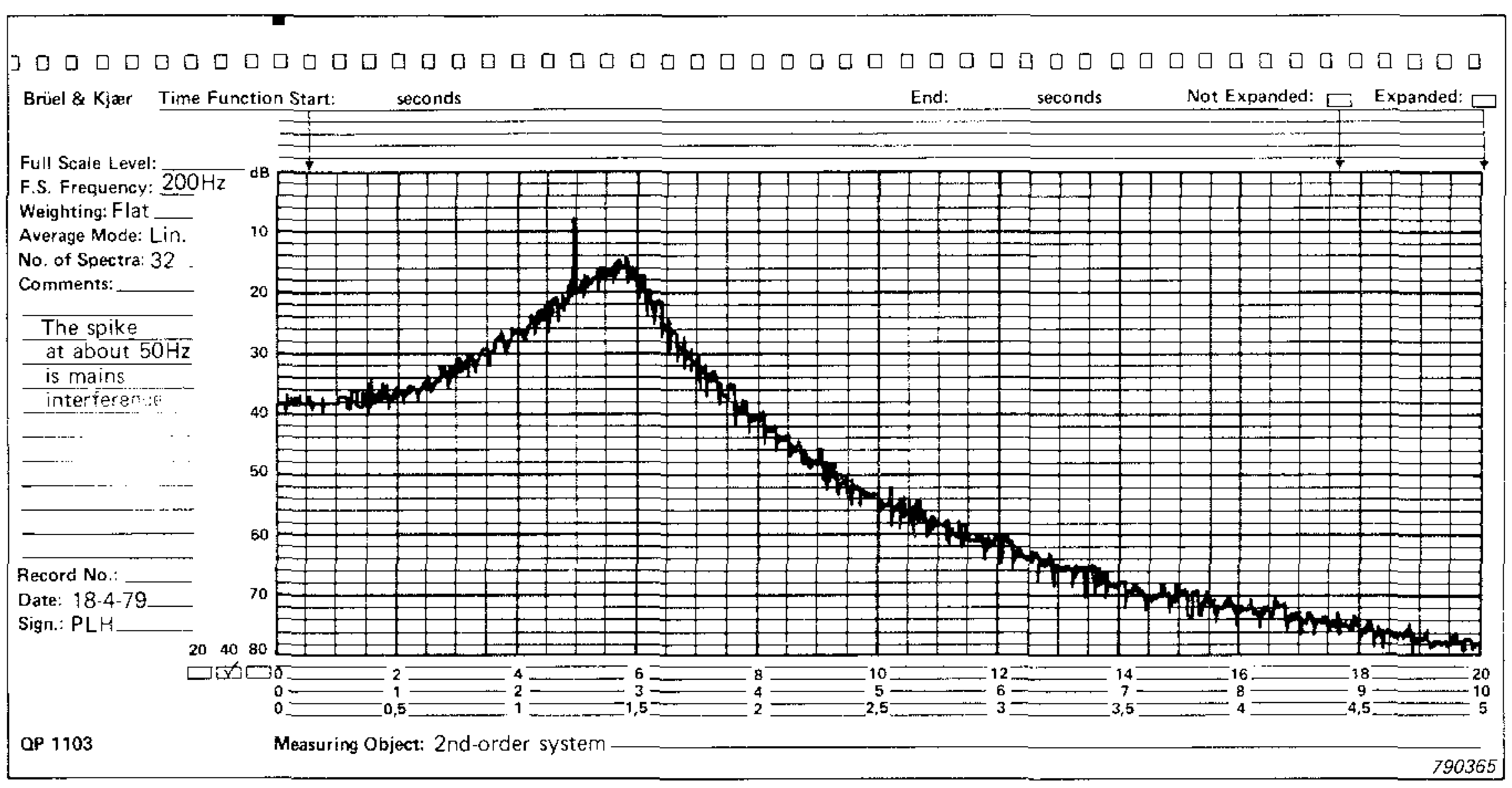

Fig.29. Example of a frequency response recording made using an equipment arrangement similar to that shown in Fig.28. Note the linear frequency scale

nals are used for deducing frequency response, the transients will be either sporadically repetitive, or capable of being generated on demand. In this case the measurements may be taken by connecting a real-time frequency analyzer to the input and output ports in turn, and determining the frequency spectrum of a number of transients in each connection. A suitable arrangement is illustrated in Fig.28, and an example given in Fig.29. The analyzer used is a 400-channel instrument utilizing Fast Fourier Transform and digital RMS detection techniques. This analyzer provides a facility for automatically subtracting a stored spectrum from a current spectrum, thereby providing a frequency response directly, as shown in Fig.29.

A continuous non-stationary signal is a signal whose frequency spectrum is changing all the time. For deducing the frequency response of a component or subsystem fed with such a signal, the solution is to record the input and output signals on a two-channel tape recorder and use a real-time analyzer to determine the average frequency spectrum of each channel on the same section of tape. Fig.30 illustrates the instrumentation needed.

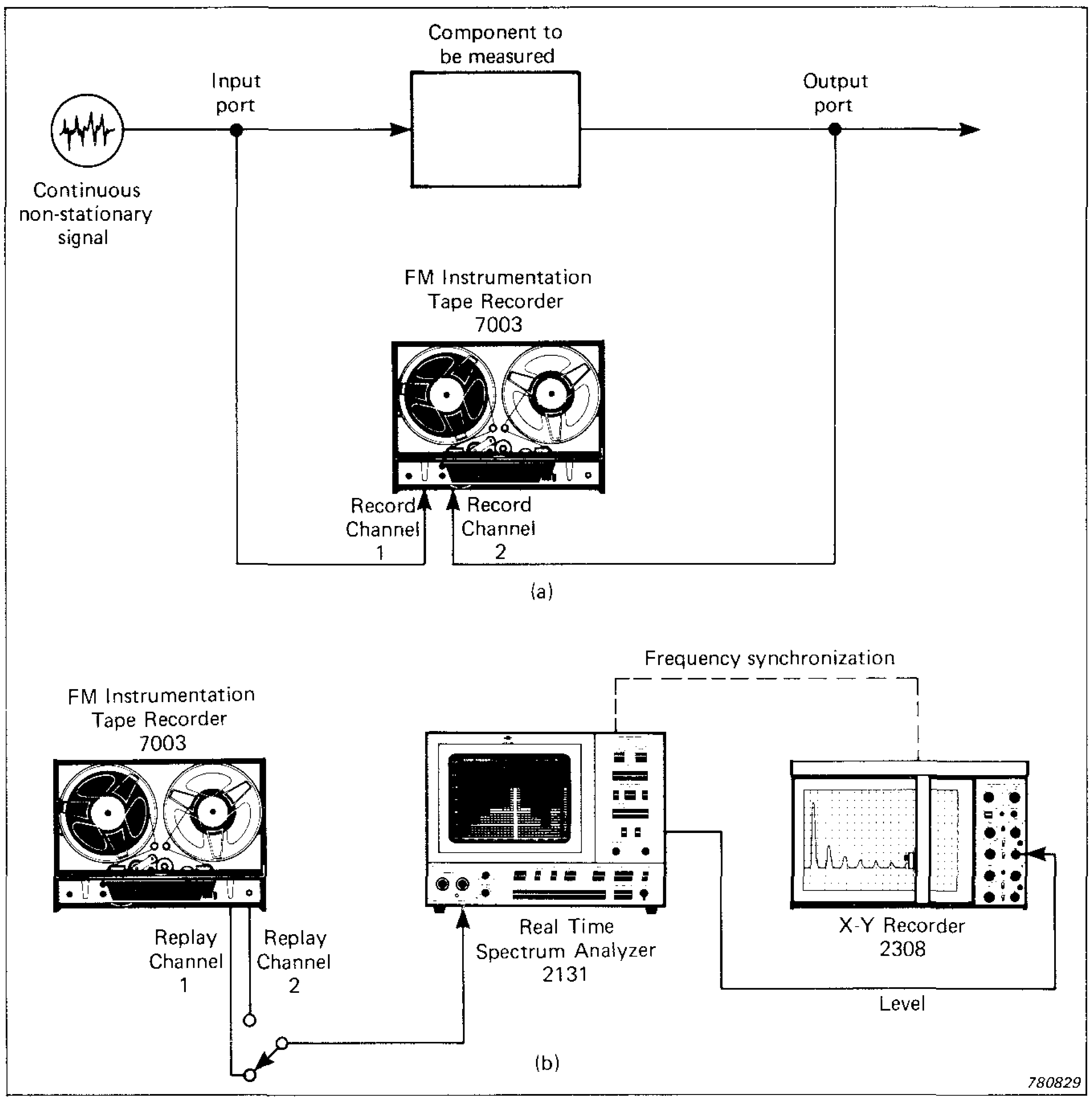

Fig.30. Measurement of component frequency response by frequency analysis of existing continuous non-stationary signals

Response Measurements on very Slow Components

Large industrial machinery incorporated in a linear control loop may prove to have such a slow response that sinewave response measurement is unpractical, either because it takes too long, or because a sine generator of sufficiently low frequency is not available. In this situation a suitable method of determining the response is to record the signals at the input and output ports of the component to be measured, using a digital event recorder, and to replay the recordings at a much faster rate, repetitively, through a tunable band-pass filter. A digital event recorder is an analogue serial-access memory utilizing digital techniques to permit a very wide operating speed range. For this application its feature is that it permits a very slowly-changing signal to be recorded in real time and reproduced at a rate fast enough for frequency analysis by practical filters and detectors. It is reproduced repetitively so that the filter may be tuned to successive frequency bands, and the detector may settle to a steady value. An arrangement of equipment for measurement with repeatable transient signals is shown in Fig.31.

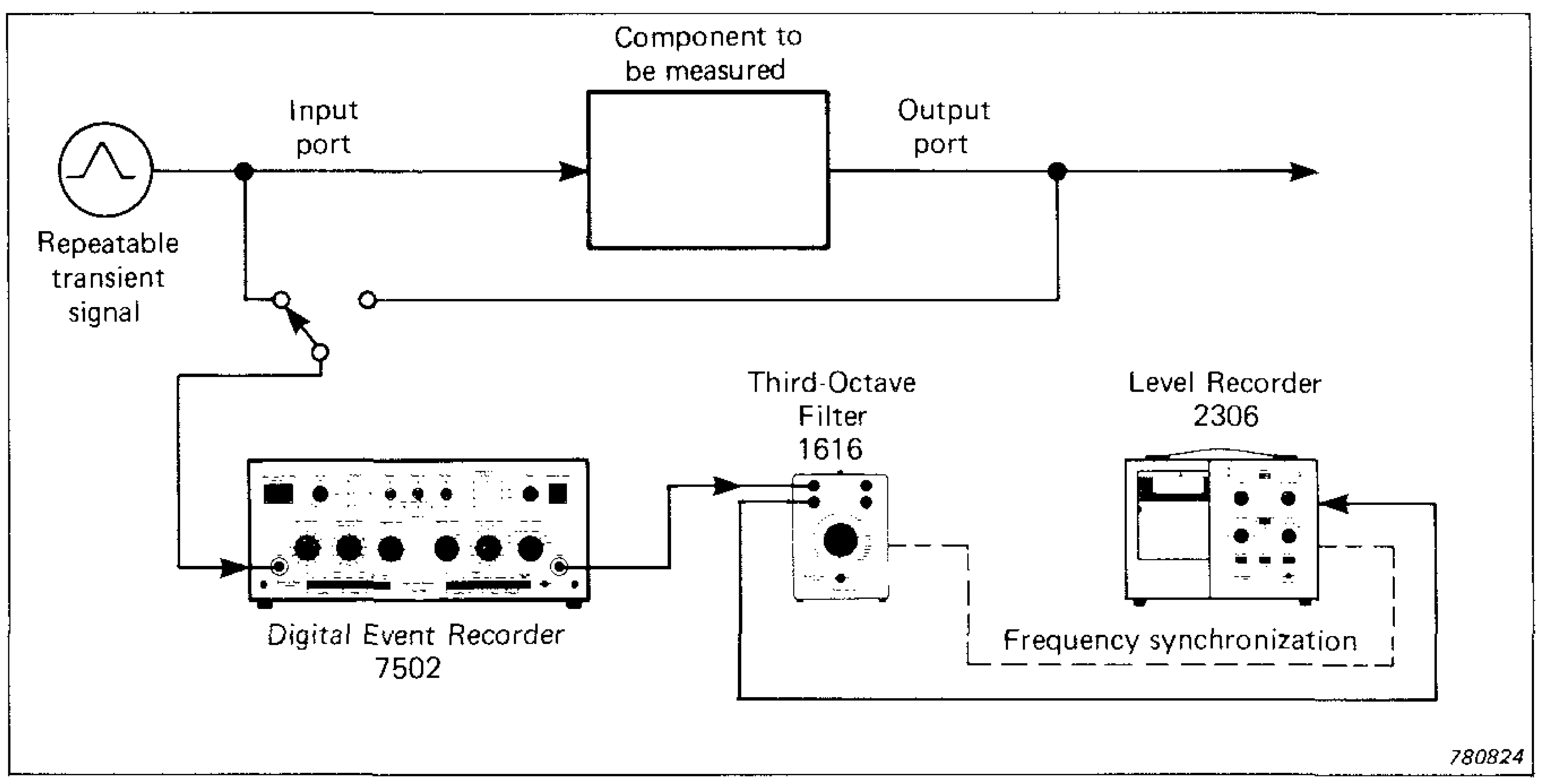

Fig.31. Response measurement on very slow components or subsystems

If the signals cannot be repeated readily, then it is necessary to make simultaneous recordings at the input and output port using two synchronized digital event recorders. The sampling nature of this measurement method restricts the bandwidth measurable in one pass to less than three decades.

Non-linear Components and Subsystems

All practical control system components are ultimately nonlinear. Nonlinearity manifests itself in the generation at the output port of energy at specific frequencies not present at the input port, including, sometimes, zero frequency. These frequencies may be distinguished from internally generated noise by their deterministic relationship with the input frequency or frequencies. It can happen, however, that internal nonlinearity generates energy at frequencies related to both the input signal and internally generated noise.

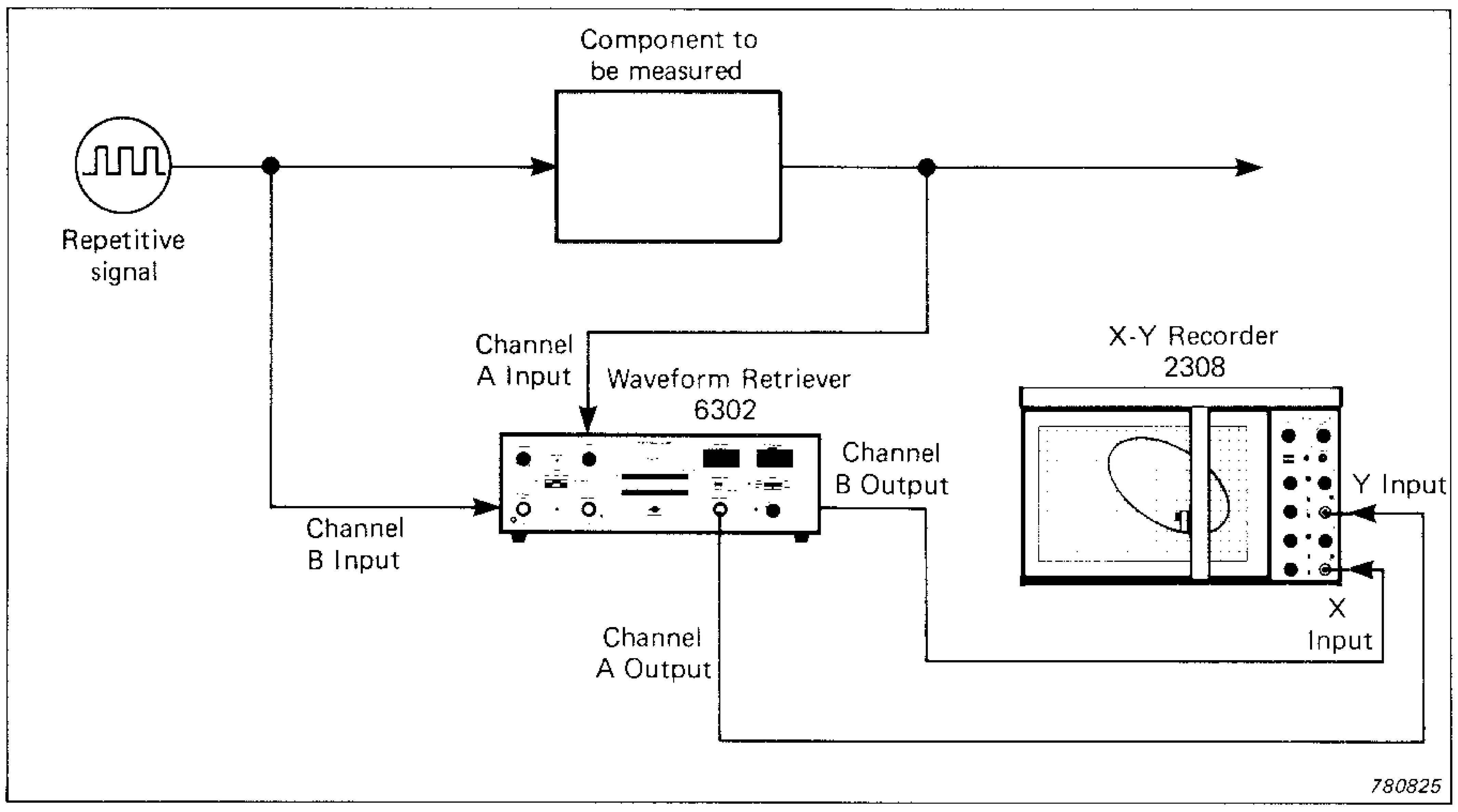

Fig.32. The use of a waveform retriever to plot input-output characteristics

Examples of nonlinear components are as follows:

1. Electronic amplifiers are usually linear (to a good approximation) up to the limit of their input rating, and for greater amplitudes they 'clip' the signals, giving no increase in output.

2. Electrical machines are linear while the flux in their magnetic circuits is air-gap controlled, or saturating, but for intermediate flux values their characteristics change appreciably with excitation level.

3. Springs and hydraulic dampers used in mechanical systems can be given all kinds of nonlinear 'constants' by their designers. As springs especially are response-determining components, the use of nonlinear springs results in a level-dependent frequency response.

4. Mechanical linkages and gear trains can suffer from backlash, which introduces hysteresis (or deadbands) into the input-output characteristic.

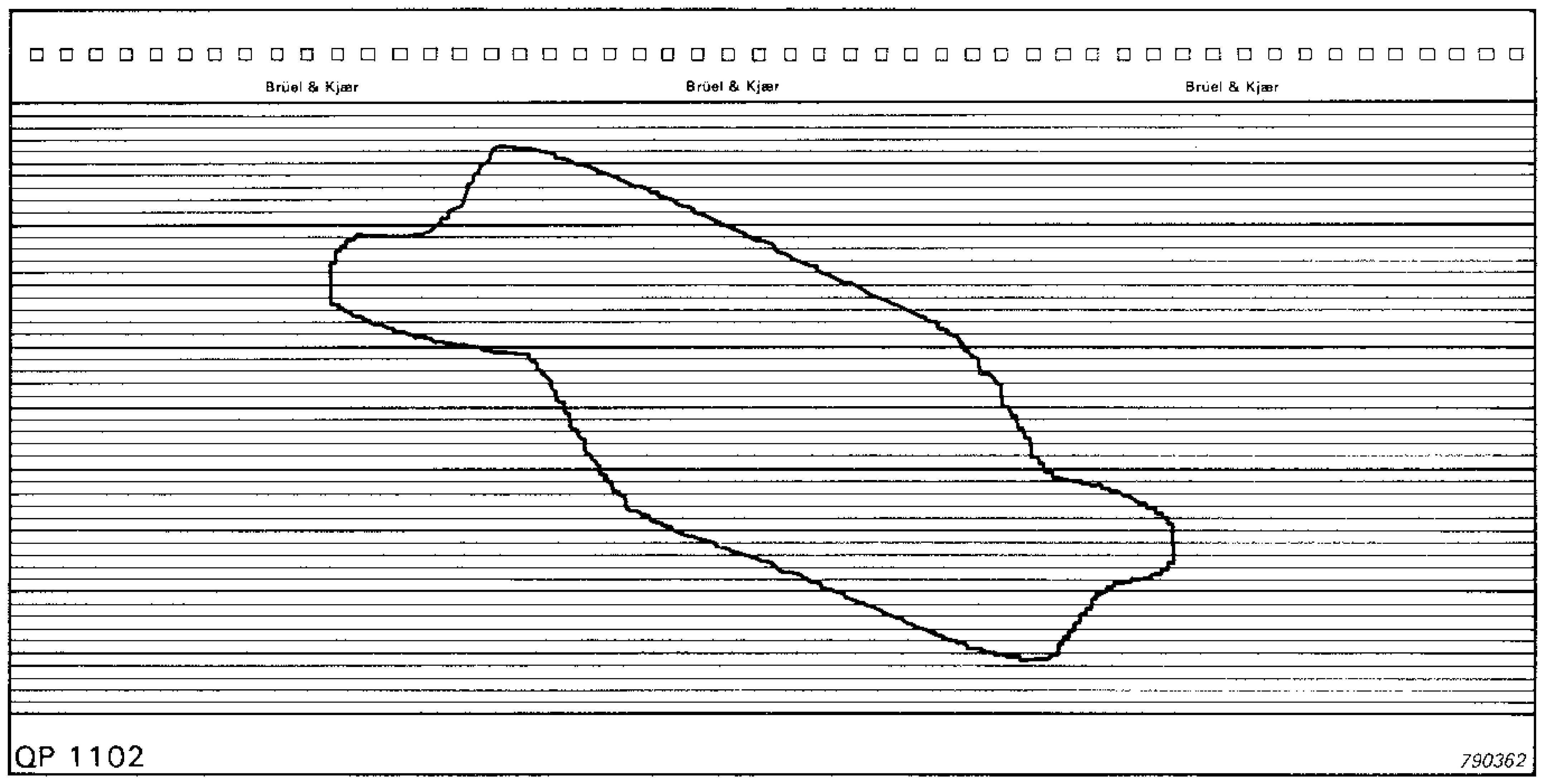

Fig.33. Example of a Cartesian recording of the input-output characteristic of a nonlinear system, made using an arrangement similar to Fig.32

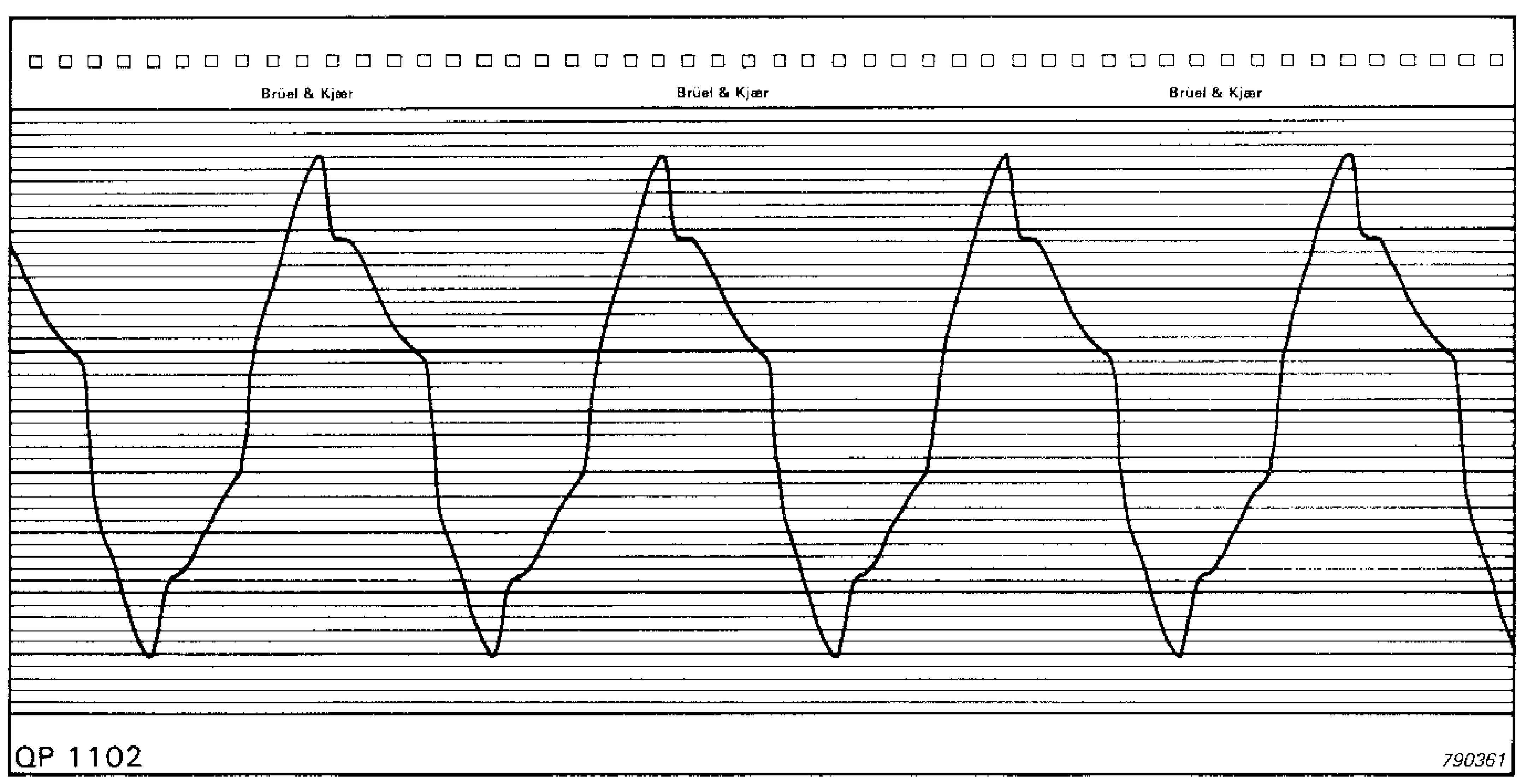

Fig.34. Output waveform corresponding to Fig.33

The amplitudes of frequencies at integral multiples of the input sinewave (harmonics) may be measured by using a tracking filter incorporating a tracking frequency multiplier, where it is required to observe the variation of harmonic generation with frequency.

The nonlinearity may be observed using a very simple arrangement like Fig.5. An X-Y display obtained in the same manner as Fig.6 provides great insight into the nature of the nonlinearity since it is in effect a Cartesian display of the input-output characteristic. An accurate graphic record of this characteristic at various frequencies may be made on an X-Y recorder if the frequencies are low enough for the pen to follow. However, in most practical applications they will not be, and a waveform retriever should be used to interface the component to be measured to the recorder. A waveform retriever is a specialized sampling device which permits repetitive signals to be recorded cleanly on a pen-recorder even when the signal is too fast for the pen and too noisy for a clear oscilloscope display to be obtained. The arrangement of equipment is shown in Fig.32, and an example given in Figs. 33 — 36.

Many practical components are nonlinear, but in the majority of cases linear design based on frequency response analysis can nevertheless be successfully employed provided the nonlinearities are measured and allowed for.

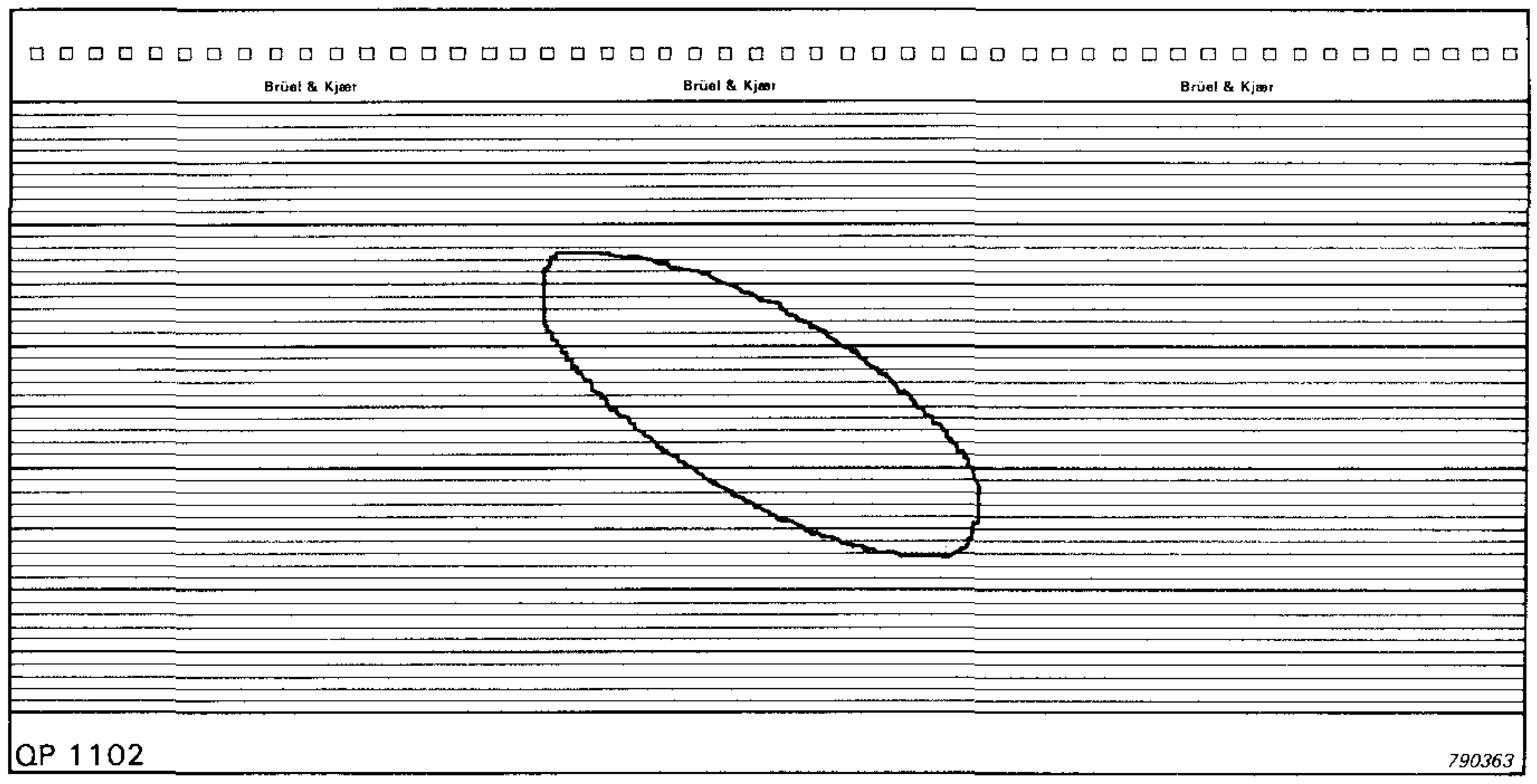

Fig.35. The same as Fig.33 but with the excitation reduced until the nonlinear effects cease to be apparent

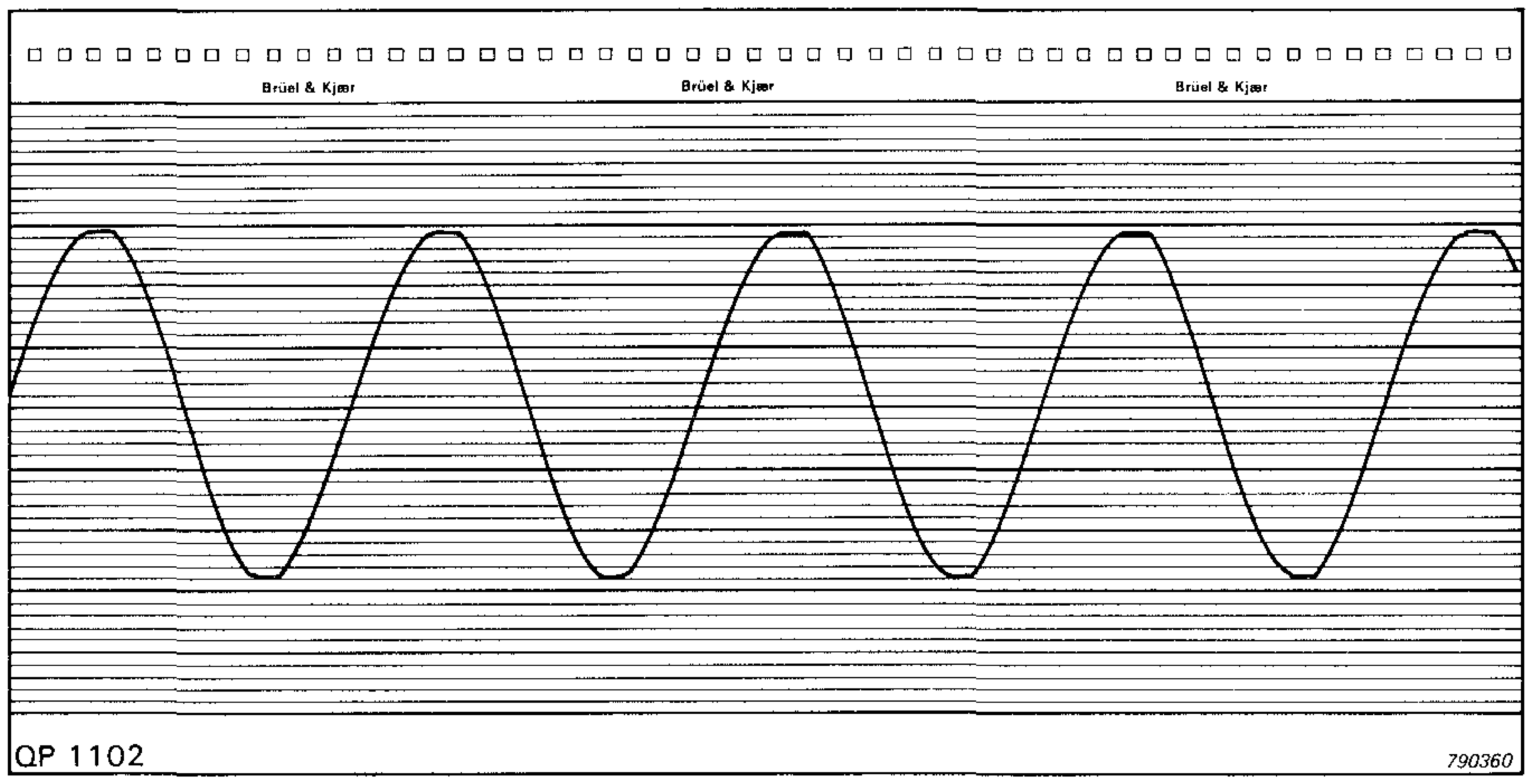

Fig.36. Output waveform corresponding to Fig.35

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Almost all the instruments mentioned in this Note are available from Brüel & Kjær or their agents. The type numbers of suggested instruments are shown in the diagrams, but in many cases alternative types may prove more suitable. For advice on the choice and use of instrumentation, and more detailed information about the instruments, please contact B & K at Nærum or your nearest representative.

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