Application Note

Gearbox Analysis using Cepstrum Analysis and Comb Liftering

by *N. Johan Wismer, Brüel & Kjær, Denmark*

Cepstrum analysis has many applications: for example echo detection and removal, or speech analysis, but the most important one in industry is related to machine diagnostics, where its ability to detect periodicities in the spectrum is taken advantage of. The Dual Channel Analyzer Type 3555 has a number of editing features that make it uniquely well suited for identification and separation of harmonic families using cepstrum analysis.

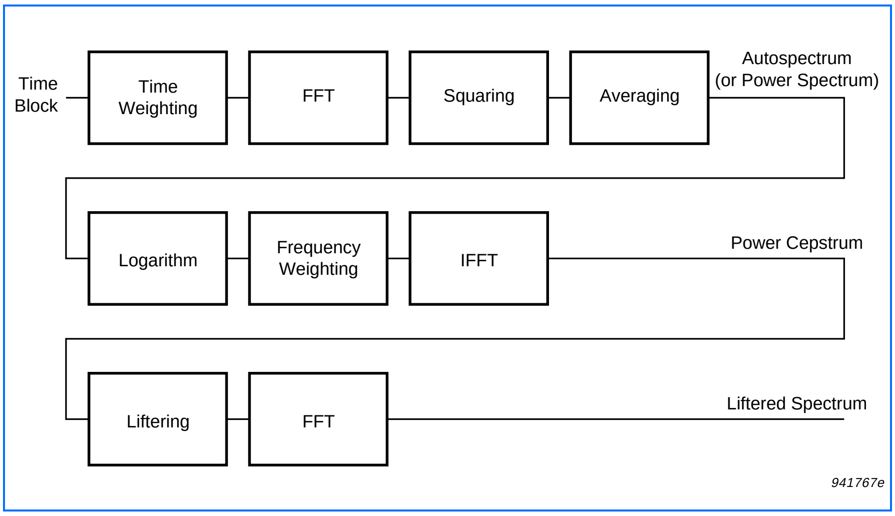

Fig. 1 Block diagram showing the data flow when performing cepstrum analysis

Introduction

The spectrum of a gearbox signal will usually consist of a number of harmonic families. These harmonic families originate from the different shafts and ball-bearings in the gearbox, and from the tooth meshing frequencies of the gears. The gears usually have numbers of teeth equal to prime numbers. This is an advantage as it causes wear to be spread out more evenly on the teeth of the gears, but it is also an advantage from a measurement point of view, as it means that the different harmonic families will usually not overlap. On the other hand, there can often be several harmonic families, and it can be difficult to separate them in the spectrum. Cepstrum is a practical tool that makes it easy to find these different harmonic families, and the individual families can be monitored for changes that might indicate that something is wrong.

How does it work?

The power cepstrum is defined as the "inverse Fourier transform of the logarithmic power spectrum", or using symbols:

$$C_{xx}(\tau) = F^{-1}\{\log G_{xx}(f)\}$$

where $G_{xx}(f)$ is the autospectrum (power spectrum).

The cepstrum can be edited, or liftered as it is called (paraphrasing of "filtered"). The equivalent spectrum, called the Liftered Spectrum, can be found by applying an FFT to the liftered cepstrum.

The data flow is shown in Fig. 1.

The cepstrum and the auto-correlation are closely related. The main difference is that the inverse FFT is performed on the *logarithm* of the power spectrum, as opposed to the power spectrum itself. The auto-correlation is mainly dominated by the highest values of the spectrum. The logarithm used when computing the cepstrum causes it to take lower level harmonics more into account than does the auto-correlation. It also means that the auto-correlation is strongly influenced by the shape of the time signal, whereas the cepstrum mainly reacts to the harmonics present in the autospectrum and much less to their relative size.

The Measurement

A spectrum averaging measurement was performed on a single stage gearbox (input shaft, one gear, output shaft). The vibration signal was measured by, and analysed on, a Dual Channel Signal Analyzer Type 3555.

When we look at the autospectrum of the signal, we see a “forest” of harmonics (see Fig. 2).

The harmonic families can be found manually by using the harmonic cursor, but this can be a tedious process. On the graph a harmonic family with Δf = 123.75 Hz has been found. The process of finding the harmonic families is much easier in the cepstrum domain (see Fig. 3).

On the graph the rahmonic (paraphrasing of harmonic) family with Δt = 8.083 ms has been found with the harmonic cursor (marked with A). Another family with a spacing of just under 20 ms can also be seen to exist (marked with B).

The 8.083 ms family corresponds to the harmonic family previously found in the autospectrum. The harmonic and rahmonic spacings are reciprocals of each other. An Auxiliary Information (ΔFREQ) performs this calculation. It reads out the equivalent frequency spacing, in this case 123.71 Hz, which is very close to the 123.75 Hz found previously.

An important feature of the 3550 analyzer, is that one or two rahmonic families can be removed from the cepstrum. To do this, we use a feature called comb liftering. In Fig. 4 the rahmonic family found previously (Δt = 8.083ms) has been removed. Note that comb liftering of cepstra can also be used when echoes are to be removed from a signal.

The cepstrum has one remaining rahmonic family with a spacing of 19.6 ms, which means that the spacing of the harmonic family must be 51 Hz (= 1/19.6 ms), which is exactly what ΔFREQ reads out. The edited cepstrum can now be transformed back to the frequency domain. This power spectrum is called a Liftered Spectrum.

In Fig. 5 the upper graph shows the liftered spectrum from which the 123.75 Hz harmonic family has been removed. The bottom graph shows the liftered spectrum where the 51 Hz family has been removed. These two harmonic families seem to be the only ones present in the measured signal.

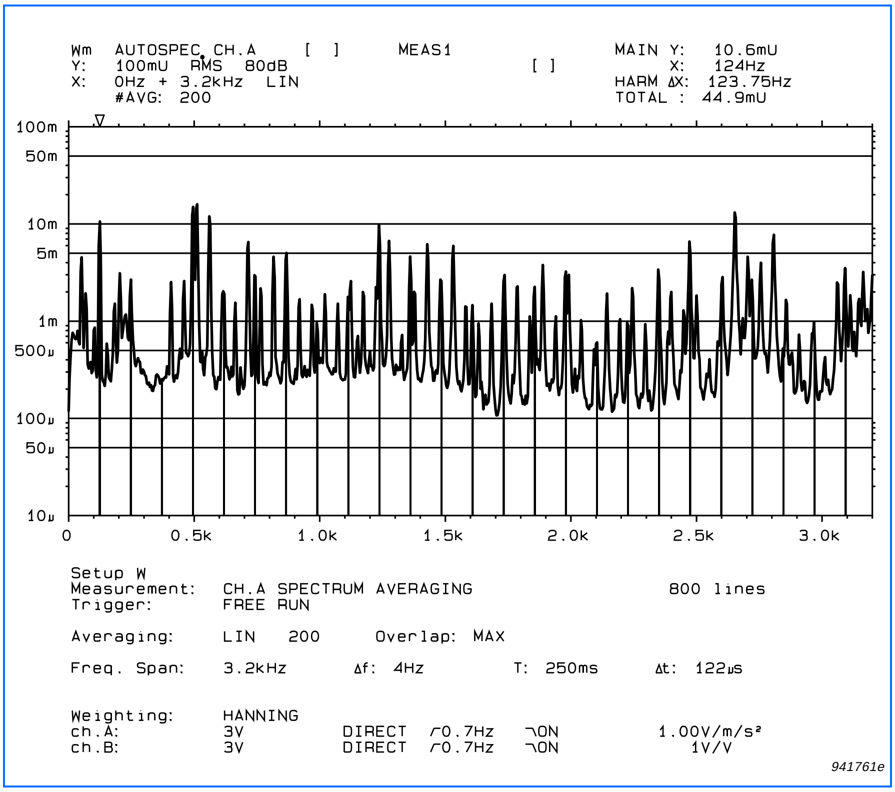

Fig. 2 The power spectrum of a gearbox vibration signal

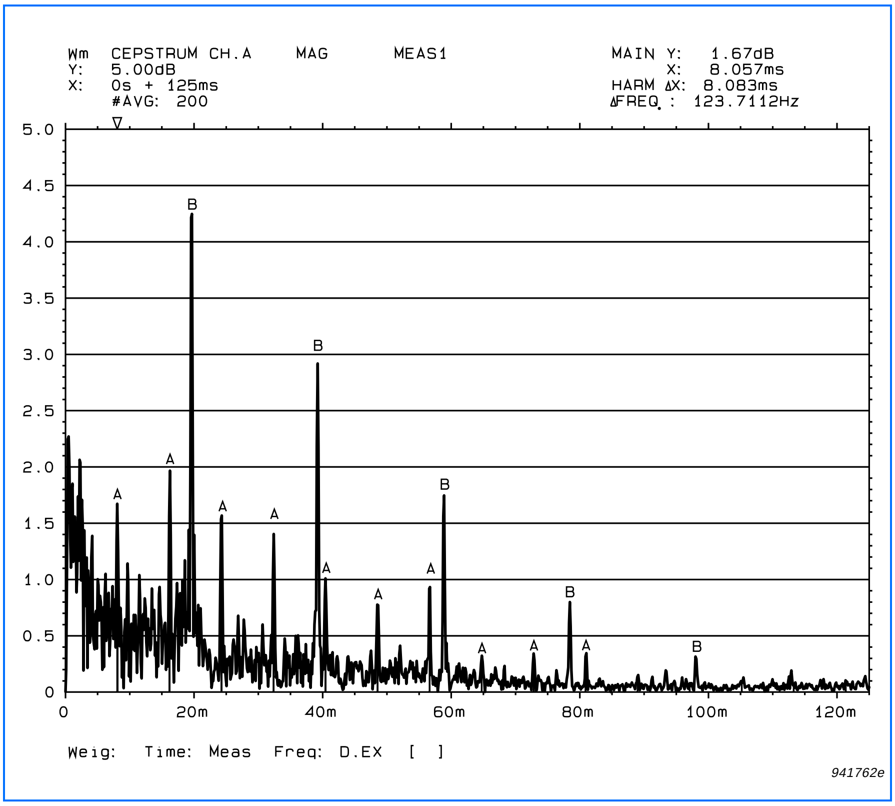

Fig. 3 The cepstrum of the gearbox vibration signal

Finally, the power present in the harmonic families can be calculated. To do this, the User-Definable Auxiliary Information (UDAI) shown in Fig. 6 will be needed. This is a small program that was entered via the analyzer's keyboard. It can also be stored on disk for later use.

In Fig. 7 the HARM_RMS UDAI is used to calculate the RMS value of the two harmonic families. Note that the computations must be performed on the autospectrum, and not on the liftered spectrum. The RMS levels of the liftered spectrum can normally not be relied on.

The RMS value of the 51 Hz family is 0.03595 m/s^2, and the RMS value of the 123.75 Hz family is 0.02629 m/s^2. In Fig. 2 the total RMS of the measured signal was found to be 0.0449 m/s^2. This means that 64.1% ($0.03595^2/0.0449^2$) of the signal power was present in the 51 Hz family, and 34.3% ($0.02629^2/0.0449^2$) of the power was present in the 123.75 Hz family. The sum of these two, 64.1% + 34.3% = 98.4%, is so close to 100%, that almost all the power present in the signal has been accounted for by the two harmonic families.

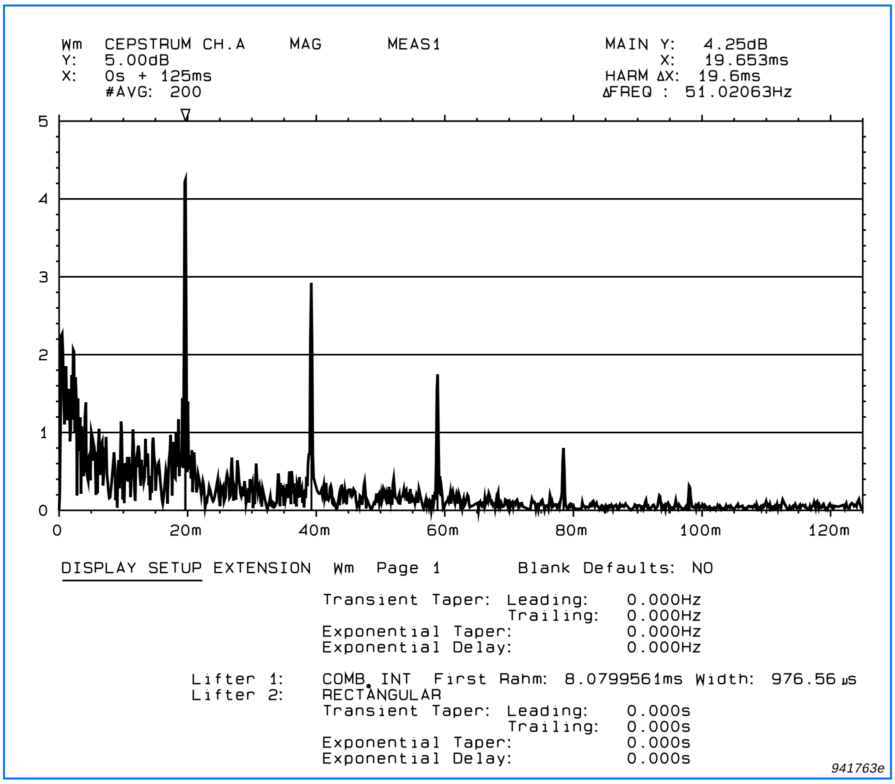

Fig. 4 Cepstrum with the 8.083 ms rahmonic family removed

Advanced Cepstrum Analysis

The Type 3550 possesses some more features that allow for other, more advanced, types of cepstrum analysis:

❍ You can combine order tracking with cepstrum analysis.
If the gearbox you are measuring on is not running at a constant speed, then an order tracked measurement can be performed. Using this feature requires that you input a tacho signal into one of the analyzer's inputs.
To be able to perform order tracking measurements, Extended Analysis Software Type 7639, or Tracking Analysis Software Type 7670 must be installed.

❍ You can perform cepstrum analysis on time capture data.
If you are measuring on a multistage gearbox, then there might be so many harmonic families that they get all mixed. Using the time capture feature, which allows for very large FFTs, will increase the frequency resolution by up to a factor of 32, and will therefore make the separation of the harmonic families easier.

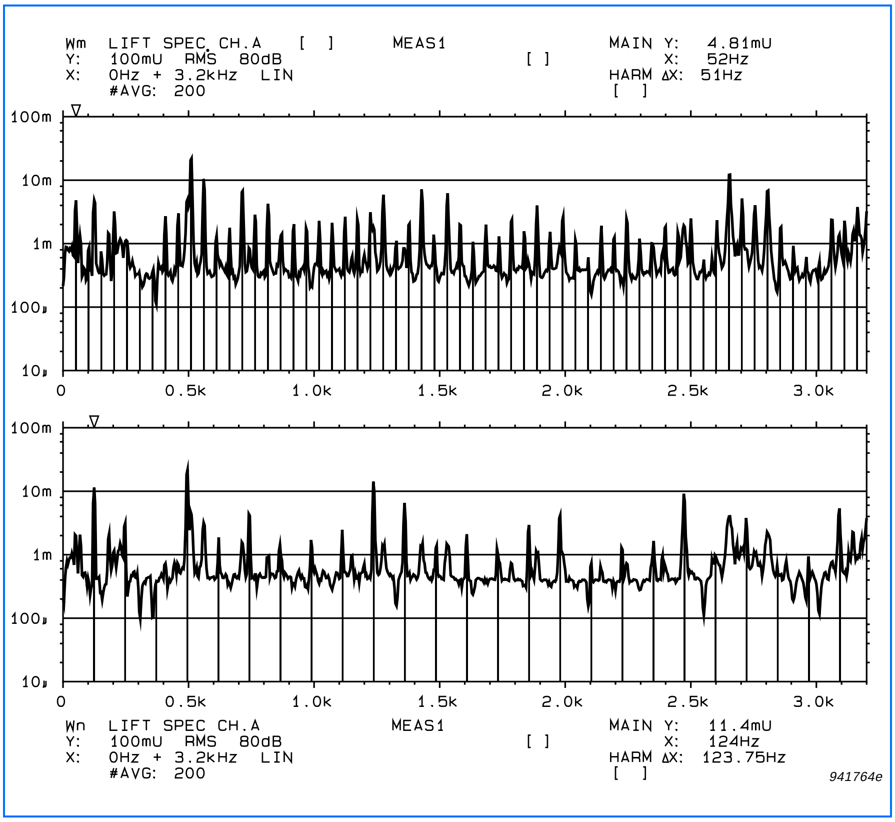

Fig. 5 Liftered spectrum with 123.75 Hz harmonic family removed (top), and with the 51 Hz harmonic family removed (bottom)

To be able to perform time capture measurements, Extended Analysis Software Type 7639, or Signal Analysis Software Type 7671 must be installed.

Finally these two features can be combined, if you want to perform cepstrum analysis on a multistage gearbox not running at a constant speed.

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USER-DEFINABLE AUXILIARY INFORMATION: HARM_RMS

Reading: ABSOLUTE                                          Reading Unit: U

Harm_rms        = sqrt ( 2/3 * sum ( Harmonics ))
Harmonics       = Harm_Mask ( Wnd_Disp_Func )
Harm_mask (x)   = select ( mask (x), zeros [x], x )
Wnd_Disp_Func   = window ( Disp_Func, Width, spectral_lines - width )
Mask (x)        = 1, + width - rem ( ramp [1, x, x] +width/2-1, Harmonic )
Disp_Func       = data_source: func
WIDTH = 3.000 ;
Rem (x, y)      = x - y * trunc ( x/y )
Harmonic        = Width_of_delta_x
                                                                  941765e
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Fig. 6   User-definable function used to calculate the RMS value of a harmonic family

Conclusion

The Type 3555 is the only stand-alone analyzer able to perform all the computations required to do advanced cepstrum analysis. We have seen that the cepstra can be edited, and that the edited cepstrum can be transformed back to a power spectrum. Editing can also be performed on frequency spectra before the cepstrum is calculated, in order to analyze frequency ranges with either harmonic families or sideband families. It has also been mentioned that cepstrum analysis can be performed on order tracked data, or on time capture data. Tracking is useful if the gearbox does not run at a steady speed. Time capture is useful if there are many harmonic families to separate.

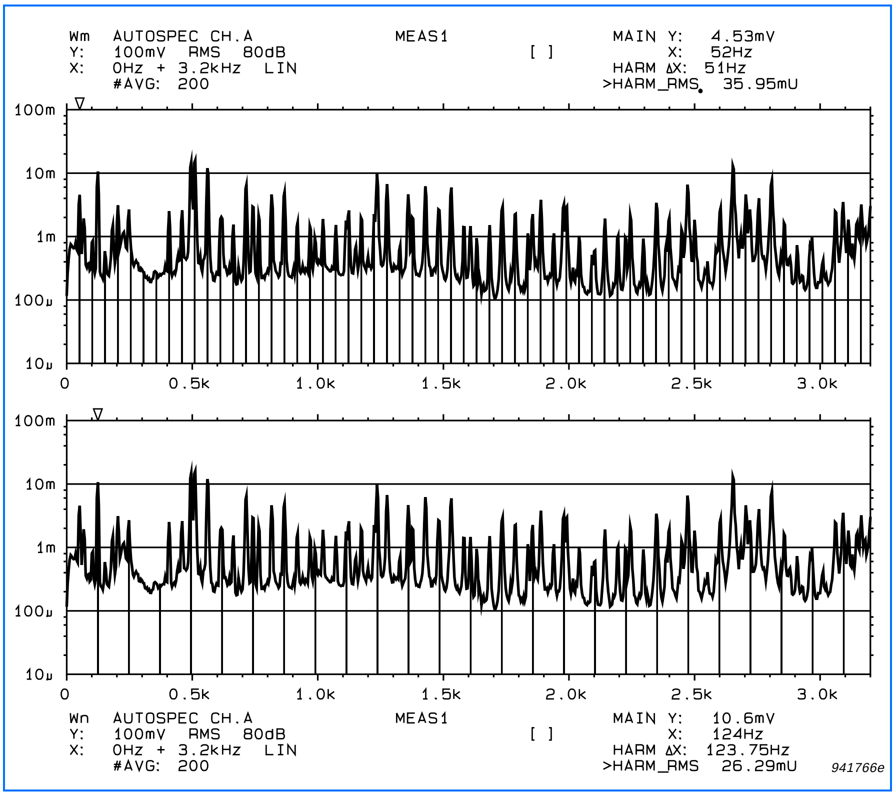

Fig. 7   The RMS value of the two harmonic families

References

[1] Brüel&Kjær, Frequency Analysis, Chapter 8, 1987 revision (BT 0007)

[2] Brüel&Kjær, Technical Review, No. 3 – 1981, Cepstrum Analysis

Brüel & Kjær

WORLD HEADQUARTERS:
DK-2850 Naerum · Denmark · Telephone: +45 45 80 05 00 · Fax: +45 45 80 14 05 · Internet: http://www.bk.dk · e-mail: info@bk.dk
Australia (02) 9450-2066 · Austria 00 43-1-865 74 00 · Belgium 016/44 92 25 · Brazil (011) 246-8166 · Canada: (514) 695-8225 · China 10 6841 9625 / 10 6843 7426
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Hungary (1) 215 83 05 · Italy (02) 57 60 4141 · Japan 03-3779-8671 · Republic of Korea (02) 3473-0605 · Norway 66 90 4410 · Poland (0-22) 40 93 92 · Portugal (1) 47114 53
Singapore (65) 275-8816 · Slovak Republic 07-37 6181 · Spain (91) 36810 00 · Sweden (08) 71127 30 · Switzerland 01/94 0 09 09 · Taiwan (02) 713 9303
United Kingdom and Ireland (0181) 954-236 6 · USA 1 - 800 - 332 - 2040
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